

Proposed Changes to the Pointing Model for the
140-ft Telescope

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August 24, 1992

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Abstract

After investigating the 140-ft pointing model, and developing new fitting software, I was able to determine that our existing pointing model is probably in error and that my proposed model fits pointing data better. Some suggestions are given as to how to further investigate the pointing problems of the 140-ft and as to what observers should do to ensure good pointing of the telescope.

1. Introduction

The pointing model used by the 140-ft control system has gone through numerous alterations over the years -- the history of these changes can be ascertained by going through old reports and memos (Pauliny-Toth, 1969; Herrero, 1972; Gordon et al., 1973; von Hoerner, 1975; von Hoerner, 1976a, 1976b, 1977). The currently used pointing model is almost identical to that proposed by von Hoerner (1976b, 1977) and which I will label the von Hoerner model.

Recently, pointing problems with the 140-ft have nudged me to reinvestigate the pointing model and the algorithms we use to fit data to the model. Although I am still investigating the pointing problems, the new algorithm I have recently developed is better suited to investigating pointing problems and is, statistically, more correct. Using the new algorithm, I have determined that some terms in the old pointing models need not exist while others terms which have been either ignored or were unknown should be added.

I will first briefly describe the various versions of the von Hoerner model that have been used since 1977 (section 2) and possible problems with those models (section 3). Next, I will present my proposed model and the fitting algorithm I have used that makes experimenting with pointing models simple (section 4). In section 5, I report the results of the new fitting model and compare them with those produced by the old models. Finally, in section 6, I discuss the implications of my research; I also give suggestions on how to further investigate the existing pointing problems and how observers can ensure good pointing of the 140-ft.

2. Old Pointing Model

In the following description of the pointing models for the 140-ft, I use the following definitions:

H, HA = Hour Angle
D, Dec = Declination
L = Latitude
Z = Zenith distance

2.1 The von Hoerner model (1977 - early 1980's)

The von Hoerner model (1976b, 1977) is based on how known physical errors or features in the telescope can produce predictable pointing error. It is, therefore, a theoretical model and not an empirical one. The Dec and Ha pointing models he suggested are:

$$\begin{aligned} \Delta \text{Dec} = & C1 + C2*\sin H + C3*\cos H + C4*(\sin D*\cos H - \tan L*\cos D) + \\ & C5*Q*(\sin L - \sin D*\cos Z)/\cos D \end{aligned} \quad (1)$$

$$\begin{aligned} \Delta H = & C6 + C7*\sin D + C8*\cos D + C9*\sin H + C10*\sin D*\sin H + \\ & C11*\cos D*\sin H + C5*Q*\cos L*\sin H + C2*\sin D*\cos H \end{aligned} \quad (2)$$

The factor Q is a weather dependent term which has the definition:

$$Q = K / [\cos Z + 0.00175*\tan(Z-2.5)]$$

where $K = 0.354 P/T - 0.0585*P_w/T + 1701*P_w/(T**2)$; P = atmospheric pressure; T = atmospheric temperature; and P_w = water vapor pressure

He ignored three terms with physical causes, two in Dec ($Q1*\sin D$ and $Q2*\cos D$) and one in HA ($Q3*\cos D*\cos H$) for which the coefficients, he ascertained, were probably close to zero. The various terms have the physical causes outlined in Table 1.

Note that two coefficients, C2 and C5, are common to both the Dec and HA models; their definition is a major difference between his model and the one I propose.

The coefficients C1, C6, and C8 should change with each receiver installation since they depend upon the repeatability in which receivers are mounted to the telescope. The changes in the three coefficients are hopefully corrected for by the process of finding the three global pointing corrections we call the 'PVLS' (see "Computer Assisted Observing with the 140-ft", pp 12-13 for details about PVLS). Usually, the operator or Friend-of-the-telescope, after every receiver installation, determines values for the PVLS by pointing on three to four sources; the determined values are then used to alter the values of C1, C6, and C8 that are stored in the Honeywell H316 computer.

The C4 and C9 coefficients will depend on whether the telescope is used at prime or Cassegrain focus and whether at Cassegrain focus the lateral focus mechanism is in or out of use. We, thus, should expect three different set of values for C4 and C9 which depend upon the optical configuration of the telescope.

2.2 Harry Payne's C12 term (early 1980's - 1987)

In the early 1980's, Harry Payne added the term

$$C12 * \cos H * \cos Z.$$

to the HA part of the von Hoerner model. There is no known physical reason for the term but Harry added it since he said it 'looked' like the term reduced the residuals of the fit.

2.3 My C12 term (1987-present)

When I looked closely in 1987 at pointing data, plus after studying the von Hoerner memos, I noticed that Harry's C12 term was closely related to the Q3 term originally ignored by von Hoerner. When I replaced Harry's C12 term with

$$C12 * \cos H * \cos D$$

the residuals from the fit were similarly reduced. The new term, unlike the Payne term, does have a physical cause (Q3 term, Table 1). Thus, between the early 1980's and today, the pointing model has had 12 terms (11 of the von Hoerner terms plus two different versions of the 12th term).

For the rest of this report, my usage of the term 'von Hoerner model' means his original eleven term model modified by the addition of

my 12th term.

3. Possible problems with the existing algorithms and model

The coefficients C1 through C12 have traditionally been found by empirically fitting pointing data to the model, although theory does predict the values for some of the coefficients.

Let me define two quantities:

$$HA(\text{predicted}) = HA(\text{catalog}) + \text{delta HA}$$

$$Dec(\text{predicted}) = Dec(\text{catalog}) + \text{delta Dec}$$

where 'catalog' means the known coordinates of a source and where delta HA and Dec are from the pointing model (eqs. 1 and 2). Some of the algorithms that have been developed perform a non-linear, least-squares fit for the coefficients C1 through C12 of:

$$\sqrt{\left\{ HA(\text{observed}) - HA(\text{predicted}) \right\}^2 + \left\{ Dec(\text{observed}) - Dec(\text{predicted}) \right\}^2} \quad (3)$$

The coefficients found by the fit should minimize the distance between where the telescope should be pointed and where it does point. Because coefficients C2 and C5 are in both the delta HA and delta Dec equations [and, thus, in HA(predicted) and Dec(predicted)], one cannot minimize HA(observed)-HA(predicted) and Dec(observed)-Dec(predicted) separately.

We can assume that the measured quantities HA(observed)-HA(predicted) and Dec(observed)-Dec(predicted), have errors which have a Gaussian distribution. However, in equation 3, we fit the quadratic sum of these two quantities and the net result is that we are not fitting an observed quantity that has errors which have a Gaussian distribution. Thus, we should have been using the more-general maximum likelihood method of fitting as opposed to the least-squares method whose basic assumptions we are violating. The coefficients that have resulted from using these non-linear, least-squares methods probably have had reasonable values but the formal errors for the coefficients cannot be trusted. One cannot ascertain whether the found coefficients, statistically speaking, have their most likely values.

Other algorithms that have been developed (e.g., Harry Payne's POINT140 program) perform a linear, least-squares fit of:

$$HA(\text{observed}) - HA(\text{predicted}) + Dec(\text{observed}) - Dec(\text{predicted})$$

Again, because of the common C2 and C5 coefficients, one cannot fit $HA(\text{observed}) - HA(\text{predicted})$ and $Dec(\text{observed}) - Dec(\text{predicted})$ separately. The least-squares method can be used here since the fitted quantities should have a Gaussian error distribution. However, the coefficients found will be such as to minimize the sum of the HA and Dec components of the pointing error instead of the magnitude of the pointing error. The algorithm do not minimize what is usually defined as the total pointing error but minimize a related quantity.

The existing algorithms and computer programs, if we ignore their statistical blunders, were very hard to modify in case one wanted to add or delete terms from the pointing model. While this cannot be considered a flaw, it would have been nice if they the were designed so as to make experimenting or playing with the pointing model easy.

The von Hoerner model assumes that there are no other causes of pointing problems besides those listed in the Herrero memo of 1972. Although this is a very reasonable policy to take, its one that I think we must violate in order to improve the pointing model. As shown in sections 5 and 6, I think I have found additional terms to the pointing equation that do not have a presently-known physical cause.

The work described here was an effort to overcome these deficiencies in how we determine coefficients and terms in the pointing model.

4. Changes to the model and the new algorithm

I have changed the von Hoerner model in a very subtle way by making one simplifying assumptions and one hypothesis.

First, I assume that the value for the refraction coefficient, C5, in the von Hoerner model is known -- not as unreasonable assumption since all previous fits of pointing data to the von Hoerner model suggest a value of 1.02 +/- 0.02 arcmin regardless of weather conditions, observing frequency, time of year, etc. Theory also predicts a coefficient close to this value (Herrero, 1972). My assumption, if not completely correct, at most introduces a few arcsec error when pointing very close to the horizon.

My hypothesis is that there may be a term in the Dec equation that has a $\sin H$ dependence that comes from a cause other than polar axis misalignment. Or, equivalently, I could hypothesized that there is a term in the HA equation that has a $\sin D \cdot \cos H$ dependence that comes from a cause other than polar axis misalignment. If either of these

assumptions is true, then the C2 coefficient in the Dec equation should not have the same value as the C2 coefficient term in the HA equation. In essence, I am replacing one term in the von Hoerner model, a hypothesis I test in section 5 by looking at the results of fitting data to the new model.

When I make these assumptions, I can modify the von Hoerner pointing model (eqs. 1 and 2) by making C5 a constant and by replacing the C2 coefficient in delta HA with a new C13 coefficient. The new pointing equations can be written as:

$$\begin{aligned} \text{Delta Dec} = & C1 + C2*\sin H + C3*\cos H + C4*(\sin D*\cos H - \tan L*\cos D) + \\ & 1.02*Q*(\sin L - \sin D*\cos Z)/\cos D \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Delta H} = & C6 + C7*\sin D + C8*\cos D + C9*\sin H + C11*\cos D*\sin H + \\ & C10*\sin D*\sin H + 1.02*Q*\cos L*\sin H + C12*\cos D*\cos H + \\ & C13*\sin D*\cos H \end{aligned} \quad (5)$$

Now, there are no coefficients which are common between the two equations and I no longer need to add quadratically the Dec and HA pointing errors (eq. 3). Instead, I can perform two linear multi-regressional least-squares fit of the two equations to pointing data. The two equations I want to minimize are simply:

$$\text{HA}(\text{observed}) - \text{HA}(\text{predicted}) \quad (6)$$

and

$$\text{Dec}(\text{observed}) - \text{Dec}(\text{predicted}) \quad (7)$$

Because of the form of the new fitting equations, I can legitimately use the least-squares technique and believe in the formal errors in the coefficients produced by the fit.

The least-squares fitting algorithm I used is that described by Press et al. (1986). The advantage of their algorithm is that by changing a parameter provided to the algorithm, I can change which terms are to be fit and which are to remain constant. I wrapped the Press et al. algorithm in a program that makes it extremely easy for the user to experiment with what the algorithm should fit or hold constant. In addition, I freely added other terms to the pointing model in order to look for any unknown but important terms that von Hoerner may have missed. The agility with which my program can add or remove terms to be fitted is the major factor which allowed me to do the investigating I needed to do.

The full versions of equations 4 and 5 that my program can handle are:

$$\begin{aligned} \text{Delta Dec} = & D1 + D2*\sin H + D3*\cos H + D4*(\sin D*\cos H - \tan L*\cos D) + \\ & D5*\sin D + D6*\cos D + D7*\cos D*\sin H + D8*\cos D*\cos H + \\ & D9*\sin D*\sin H + D10*\sin D*\cos H + D11*\sin^2 D + \\ & D12*\cos^2 D + 1.02*Q*(\sin L - \sin D*\cos Z)/\cos D \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Delta HA} = & D13 + D14*\sin D + D15*\cos D + D16*\sin H + D17*\cos H + \\ & D18*\cos D*\sin H + D19*\cos D*\cos H + D20*\sin D*\sin H + \\ & D21*\sin D*\cos H + D22*\sin^2 H + D23*\cos^2 H \end{aligned} \quad (9)$$

(Instructions are provided with the program on how to add additional terms.) Note that I have used D's to designate my coefficients from the von Hoerner model even though some of the coefficients are equivalent to those in the von Hoerner model. Table 2 shows how my D coefficients map into to the von Hoerner C coefficients:

I can now use equation 6 and 8 to fit for the D1-D12 coefficients and equation 7 and 9 for the D13-D23 coefficients.

5. Results with the new model and algorithm

By fitting pointing data to to the pointing equations, and by using the new program which allows the user to dynamically state which set of coefficients are to be fitted and which are to be held constant, I was able to ascertain which of the D coefficients have statistically relevant values. I used data for eight pointing runs which went back to November 1990 and which were at frequencies that ranged from 22 GHz down to 1.6 GHz.

My first attempt was to fit for D1-D3 and D5-D23. I didn't fit for D4 at this point since it is a coefficient to a term that has a similar dependence as the combination of the D6 and D10 terms (see eq. 8). That is, I wasn't sure whether we should be using the D4 term or the more general combination of the D6 and D10 terms. If the ratio of the derived values of D6 to that of D10 is consistently close to the value of $-\tan L$, then I can assume that the D4 term is all we need to fit and we shouldn't use the D6 and D10 terms. If the ratio is consistently different from $-\tan L$, then the D4 term should not be fitted for.

Finding which coefficients had significant values wasn't too

difficult. Basically, I performed a fit on each of the eight pointing runs and produced eight sets of coefficients. By simple inspections, many coefficients were seen to be non significant -- their values for every run were much smaller than their standard deviations. I then redid the fits but specified that these coefficients were to be held constant and have a value of zero. The results of the second fit indicated that other terms were not significant, so I similarly eliminated them. The values of Chi² (the rms average values of eq. 6 and 7) did not rise significantly between the first and second fit which confirmed that the eliminated terms probably were not needed. By the third or fourth iteration, it was clear that all of the remaining terms were statistically significant. If I eliminated any additional terms, Chi² would increase substantially but if I reintroduced any previously eliminated terms, Chi² wouldn't decrease substantially. The results of the first few fits are too voluminous to provide here but the results of the last iteration are given in Table 3.

It was easy to determine that I should be fitting D4 and that the D6 and D10 terms are not needed. In addition, the values for the coefficients I found for the D5, D7-D9 and D11-D12 terms are not statistically significant but the D1-D3 terms are significant. Thus, except for the definition of D2 (related to C2), I agree completely with the Dec part of the von Hoerner model -- no terms like those I tried need to be added to and no remaining terms should be removed from the Dec equation.

The D13-D16 and D20-D21 terms are significant but the D17-D19 and D22-D23 terms are not significant. Note that the eliminated D18 and D19 terms are a part of the von Hoerner model which implies that we have had two terms in the von Hoerner's HA model that probably are not needed.

If my hypothesis in section 3 were wrong, then the values of my D2 coefficient should be statistically close to that found for D21, which is obviously not true. Thus, my hypothesis seems to be valid.

My model, once I eliminate the insignificant terms becomes:

$$\begin{aligned} \Delta \text{ Dec} = & D1 + D2*\sin H + D3*\cos H + D4*(\sin D*\cos H - \tan L*\cos D) + \\ & 1.02*Q*(\sin L - \sin D*\cos Z)/\cos D \end{aligned} \quad (10)$$

$$\begin{aligned} \Delta \text{ HA} = & D13 + D14*\sin D + D15*\cos D + D16*\sin H + D20*\sin D*\sin H + \\ & D21*\sin D*\cos H + 1.02*Q*\cos L*\sin H \end{aligned} \quad (11)$$

I then gave all insignificant coefficients a value of zero, held them fixed, and fitted for D1-D4, D13-D16, and D20-D21. The results of the final fit are given in Table 4.

In Table 5, I provide the HA and Dec rms errors produced by the von Hoerner model using Harry Payne's POINT140 program as well as the errors produced by my model and algorithm. In figure 1, I compare the residuals for a subset of the data for the old and new models. The superiority of the new model over the old is quickly apparent.

6. Results and problems with the new model

A careful investigation of Table 4 reveals certain interesting results. I will step through the various coefficients one by one and relate what I notice.

6.1 D1, D13, and D15

The D1, D13, and D15 coefficients (equivalent to C1, C6, and C8 in the von Hoerner model) are supposed to vary from one receiver installation to another (i.e., they are related to the PVLS) but, amazingly enough, the scatter in the values for these coefficients suggest that we mount are receivers in a somewhat consistent fashion.

The large standard deviations I found for the value of the coefficients for a large number of data points suggest that our determination of PVLS after a receiver installation with only three or four measurements cannot be very accurate. It isn't clear to me whether our practices in determining PVLS improves the pointing or whether we would be better off to simply set the PVLS to zero and always use the same values for D1, D13, and D15.

A clue as to why we find large standard deviations comes from a glance at the covariance matrix produced by the fit. The matrix indicates a strong correlation between the D13 and D15 terms which suggests that one cannot determine very well D13 and D15 separately but that D13+D15 should have a well determined value. I suggest that we investigate whether our pointing would be better if we set $P2 = 0$, only fitted for P1, and applied a correction only to D13.

In addition, we should look into whether or not it is practical to take great care in mounting receivers. Then, we could possibly find a set of values for the coefficients that would be better than our error-prone determinations of PVLS.

More work is required to determine which of these options is best.

6.2 D2

From Table 4, it is apparent that the value for the D2 coefficient jumps between two values -- approximately -0.5 and -1.2. The standard deviations from the fits are so small that this must be interpreted as a real jump. Historically, D2 (equivalent to C2 in the von Hoerner model) has had a value of -0.5 so I believe the normal state of the telescope is when $D2 = -0.5$ and that something is amiss with the telescope when $D2 = -1.2$.

The jump in pointing has plagued the telescope for at least two years and we are not any closer in finding the culprit. It is noticed with the Cassegrain system but may exist at prime focus since we do not carefully monitor pointing at the lower frequencies. Pointing jumps do not seem to correlate with any particular combination of lateral focus status, usage of the beam splitter, or which Cassegrain receiver is in use. Sometimes, the simple act of remounting the subreflector after a week hiatus eliminates the problem.

According to the von Hoerner model, D2 and D21 should have the same values but, according to my results, they do not -- D21 doesn't statistically fluctuate very much, it doesn't jump when D2 jumps, nor does it have a similar value to D2. This suggests that the jump in pointing is not a change in the polar axis alignment; something else must be producing a declination pointing error of magnitude 0.7 arcmin which depends upon the sine of HA.

With the von Hoerner model, we could not adjust D2 to correct the declination errors without making the HA errors worse. With my new model, we have the freedom to alter D2 whenever we think we need to without compromising the HA pointing. I suggest that we keep a record of the circumstances when D2 apparently needs to have its value changed. I also suggest that we give D2 a default value of -0.5 and prepare instructions for the operator in case they need to alter D2 to a value of -1.2.

The need to alter D2 should be easily apparent to the operator. The change only affects the Dec pointing and has a \sinh dependence. That is, if the operator sees a difference of 1.4 arcmin in the Dec pointing between 6 hours east and west, for all sources at all declinations, then we should use a D2 value of -1.2. If they see an error in HA pointing, or the Dec pointing doesn't follow a \sinh dependence with the proper magnitude, then they should not alter the value of D2.

6.3 D3

D3 (equivalent to von Hoerner's C3) has very consistent values and apparently is not a problem. It also has a value very close to that

found historically. I suggest we take and use an average value of D3.

6.4 D4 and D16

D4 and D16 (equivalent to von Hoerner's C4 and C9) have consistent values when the lateral focus mechanism is in operation or when at prime focus. The values are also consistent with historical values. My current results do not provide good enough values to use for Cassegrain focus operation without the lateral focus mechanism.

I suggest that we only update the values of D4 and D16 for prime focus and Cassegrain, lateral-focus-on observations and that we try to obtain better values for the coefficients in the case of Cassegrain, lateral-focus-off observations. In the meantime, we should continue to use previously-determined values for D4 and D16 for Cassegrain, non-lateral focus observations.

6.5 D14

The standard deviations for D14 (equivalent to von Hoerner's C7) are constantly high and a look at the covariance matrix produced by the fit indicates that the determinations of the D13, D14, and D15 coefficients are correlated. The correlation is not as strong as that between D13 and D15 (discussed above), but it does mean that only a large number of data points will improve our determination of D14.

The determined values of D14 are very close to those reported historically for the 140-ft.

6.6 D20

The D20 term is equivalent to von Hoerner's C10 and has values which are very close to those reported historically. There is some correlation between D20 and D16, hence the large standard deviations.

6.7 D21

The D21 term has no equivalent in the von Hoerner model but, as described in section 4, was originally the same as von Hoerner's C2. In reality, it is a new term in the equation for which no complete physical cause is known, although Polar axis orientation probably plays some role. There is some correlation between D21 and D14, hence the

large standard deviations.

6.8 Average values for coefficients

My best estimates of the values for the various coefficients were determined by a weighted average of the values in Table 4 and are reported in Table 6.

In order to ascertain what would be the pointing errors if we use the suggested coefficients, I performed one more fit specifying that only D1, D12, and D14 were to be fitted (i.e., the PVLS terms which should differ between pointing experiments) and I held the rest constant with the appropriate values from Table 6. I am trying to mimic what an average observer should expect if: (1) we used my model and the coefficients in Table 6; (2) if we do a good job in determining the PVLS; and (3) if no other pointing is done (i.e., the observer doesn't ascertain local pointing corrections and points blindly).

The rms after the fit (Table 7) suggests that, except for D2, the pointing does not change much over the course of a few years. The changes are significant enough that our use of average values for the coefficients in the model will only provide good but not excellent pointing. Since it is impractical to perform pointing runs every few weeks, this is the best that can be done with a static pointing model.

Observers who blindly point the 140-ft (assuming that PVLS have been determined) can expect their pointing to be in error occasionally by more than 20 arcsec rms! I recommend that if observers want more accurate pointing they should perform measurements of local pointing corrections. When observing a source as it rises and sets, observers should also occasionally repeat these local pointing measurement every one to two hour so as to ensure that, barring a problem in D2, the pointing will be better than 15 arcsec rms.

7. Conclusion

The following points summarize the results of this report:

- o The new pointing model, in comparison to the von Hoerner model, provides a better fit to pointing data taken with the 140-ft telescope. The new model has the side benefit of making the associated statistics easier to handle than some of the old algorithms.
- o The new algorithm I have used allows for easily manipulating the pointing model so as to explore which terms are important and which are not. Previous algorithms have not been

completely correct due to an incorrect use of some statistical assumptions.

- o Various extra terms have been added to the model and tested against data in order to ascertain whether any previously unknown terms have been overlooked or whether existing terms are not really needed. Two terms in the von Hoerner model aren't needed while one was never included.
- o One coefficient in the new model (D2) jumps in value from time to time and some suggestions are given as to how to handle such jumps and how we can further investigate the cause of the jumps.
- o The terms in the pointing model are examined for systematic affects. Some suggestions are provided as to how to improve the determination of the coefficients in the pointing model.
- o A table is provided that gives my best estimates of the values of the coefficients in the new model. I investigate the ramifications of using these best estimates on the pointing of the 140-ft and provide suggestions as to what strategies observers can follow to ensure good pointing.

References

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Figure Captions

Figure 1: The residuals (in arcsec) for the June 1992, 22 GHz pointing
run using the von Hoerner model and Harry Payne's POINT140 program.
Each plot depicts the residuals for the indicated range of
declinations. The top row of plots show the HA residuals and the
bottom row the Dec.

Figure 2: The same as Figure 1 but using the new model and algorithm.

Table 1: Physical Causes for Terms in Von Hoerner Model

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C1 -- Dec Index Error
C2 -- Polar Axis Orientation
C3 -- Polar Axis Orientation and Fork (Mount) Flexure
C4 -- Reflector Flexure
C5 -- Refraction
C6 -- Collimation
C7 -- HA and Dec Axis Perpendicularity
C8 -- HA Index Error
C9 -- Reflector Flexure
C10 - Fork (Mount) Flexure and Polar Axis Orientation
C11 - HA Encoder Eccentricity and Fork (Mount) Flexure
    
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The ignored terms correspond to:

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Q1, Q2 -- Dec Encoder eccentricity
Q3 -- HA Encoder eccentricity
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Table 2: Mapping of Von Hoerner Coefficients to those in the New Model

Von Hoerner	New
C1	D1
.	D2 (Related to C2)
C2 related to D2 and D21	
C3	D3
C4 related to D6 and D10	D4
C5	Constant
Q1	D5
Q2	D6 (Related to C4)
.	D7
.	D8
.	D9
.	D10 (Related to C4)
.	D11
.	D12
C6	D13
C7	D14
C8	D15
C9	D16
.	D17
C11	D18
C12 or Q3	D19
C10	D20
.	D21 (Related to C2)
.	D22
.	D23

Table 3: Results of the Initial Fits

	Jan 91	Feb 91	Jun 91	Feb 92	Jun 92	Nov 90	Feb 91	Jun 92
D1	3.19 (0.45)	1.26 (0.41)	2.43 (0.68)	3.87 (0.47)	2.97 (0.27)	1.98 (0.41)	2.96 (0.35)	2.36 (0.46)
D2	-0.45 (0.10)	-1.42 (0.12)	-1.09 (0.13)	-0.28 (0.13)	2.97 (0.27)	1.98 (0.41)	2.96 (0.35)	2.36 (0.46)
D3	-2.23 (0.25)	-2.09 (0.25)	-2.95 (0.32)	-2.46 (0.31)	-2.01 (0.21)	-2.32 (0.26)	-2.53 (0.22)	-2.26 (0.28)
D6	5.18 (0.47)	5.51 (0.45)	6.80 (0.83)	5.34 (0.58)	5.53 (0.34)	-0.71 (0.53)	-0.61 (0.38)	-0.85 (0.55)
D10	-6.66 (0.26)	-6.46 (0.27)	-6.88 (0.32)	-6.77 (0.35)	-6.33 (0.23)	0.91 (0.25)	-0.03 (0.25)	0.90 (0.30)
D13	-0.89 (0.53)	-1.99 (0.46)	-1.50 (0.91)	-2.26 (0.67)	-0.87 (0.40)	-1.95 (0.63)	-1.11 (0.39)	-1.74 (0.64)
D14	-1.33 (0.47)	-1.49 (0.37)	-0.72 (0.51)	-1.21 (0.47)	-1.14 (0.30)	-1.11 (0.42)	-1.06 (0.35)	-1.28 (0.43)
D15	0.10 (0.54)	0.77 (0.49)	1.55 (0.96)	0.52 (0.71)	0.85 (0.43)	0.67 (0.68)	0.83 (0.41)	1.15 (0.69)
D16	-11.34 (0.15)	-12.40 (0.17)	-12.42 (0.19)	-11.63 (0.17)	-11.87 (0.13)	-0.16 (0.18)	-0.74 (0.12)	-0.28 (0.19)
D20	-0.42 (0.32)	-0.29 (0.29)	-0.34 (0.36)	-0.33 (0.30)	-0.29 (0.20)	-0.44 (0.28)	-0.42 (0.22)	-0.26 (0.33)
D21	0.87 (0.49)	1.32 (0.38)	-0.25 (0.49)	1.22 (0.56)	1.23 (0.29)	0.41 (0.44)	0.02 (0.38)	0.76 (0.45)
HA rms	17.6"	10.6"	15.2"	8.0"	8.6"	6.1"	12.8"	8.6"
Dec rms	12.5"	12.3"	13.8"	8.4"	14.5"	7.5"	9.6"	14.3"
Freq.	22 GHz	10 GHz	22 GHz	22 GHz	15 GHz	5 GHz	8 GHz	1.8 GHz
No. Pts.	276	207	179	130	297	167	278	141
Rx	Cassegrain	Cassegrain	Cassegrain	Cassegrain	Cassegrain	Prime Focus	Cassegrain	Prime Focus
Lat. F.	On	On	On	On	On	On	Off	On

Note: Values for D1 through D20 are in arcmin. The formal standard deviations of the fitted coefficients is given in parenthesis. The average HA and Dec residuals after the fit are given as well as the number of data points that went into the fit. Whether the receiver was a Cassegrain or prime focus, as well as whether the lateral focus tracking was on, is also indicated.

Table 4: Results of the Final Fits

	Jan 91	Feb 91	Jun 91	Feb 92	Jun 92	Nov 90	Feb 91	Jun 92
D1	3.13 (0.22)	1.50 (0.20)	3.27 (0.24)	3.86 (0.23)	3.21 (0.14)	1.98 (0.19)	2.61 (0.17)	2.30 (0.21)
D2	-0.45 (0.10)	-1.42 (0.11)	-1.09 (0.13)	-0.28 (0.13)	-1.19 (0.08)	-0.51 (0.11)	-0.46 (0.09)	-0.53 (0.12)
D3	-2.24 (0.25)	-2.07 (0.23)	-2.74 (0.28)	-2.46 (0.29)	-1.91 (0.18)	-2.32 (0.24)	-2.61 (0.22)	-2.30 (0.26)
D4	-6.63 (0.16)	-6.60 (0.13)	-7.22 (0.20)	-6.76 (0.24)	-6.53 (0.12)	0.91 (0.19)	0.23 (0.12)	0.93 (0.19)
D13	-0.89 (0.53)	-1.99 (0.46)	-1.50 (0.91)	-2.26 (0.67)	-0.87 (0.40)	-1.95 (0.63)	-1.11 (0.39)	-1.74 (0.64)
D14	-1.33 (0.47)	-1.49 (0.37)	-0.72 (0.51)	-1.21 (0.47)	-1.14 (0.30)	-1.11 (0.42)	-1.06 (0.35)	-1.28 (0.43)
D15	0.10 (0.54)	0.77 (0.49)	1.55 (0.96)	0.52 (0.71)	0.85 (0.43)	0.67 (0.68)	0.83 (0.41)	1.15 (0.69)
D16	-11.34 (0.15)	-12.40 (0.17)	-12.42 (0.19)	-11.63 (0.17)	-11.87 (0.13)	-0.16 (0.18)	-0.74 (0.12)	-0.28 (0.19)
D20	-0.42 (0.32)	-0.29 (0.29)	-0.34 (0.36)	-0.33 (0.30)	-0.29 (0.20)	-0.44 (0.28)	-0.42 (0.22)	-0.26 (0.33)
D21	0.87 (0.49)	1.32 (0.38)	-0.25 (0.49)	1.22 (0.56)	1.23 (0.29)	0.41 (0.44)	0.02 (0.38)	0.76 (0.45)

Note: The D13 through D21 coefficients are identical to those in Table 3 since the HA equations was not altered between the fits.

Table 5: Rms of Residuals for Von Hoerner and New Model

Date	Frequency (GHz)	Von Hoerner model and POINT140 program		New model and new program	
		HA rms (")	Dec rms (")	HA rms (")	Dec rms (")
Jan 91	22	24.2	21.6	17.6	12.5
Feb 91	10	16.9	11.9	10.6	12.8
Jun 91	22	32.3	22.5	15.2	15.0
Feb 92	22	12.0	10.3	8.0	8.4
Jun 92	15	18.8	17.1	8.6	14.9
Nov 90	5	7.9	7.9	6.1	7.5
Feb 91	8	13.5	10.3	12.8	10.6
Jun 92	1.8	12.9	16.1	8.6	13.4

HA and Dec rms are defined as the rms average values of HA (observed) - HA (predicted) and Dec (observed) - Dec (predicted), respectively.

Table 6: Best Values for Coefficients

Coefficient	Value and Standard Dev.	Notes
D1	2.72 (0.07)	
D2	-0.45 (0.05)	
	-1.23 (0.06)	[If a pointing jump exists]
D3	-2.28 (0.08)	
D4	-6.67 (0.07)	[Cassegrain Focus, Lateral Focus On]
	1.07	[Cassegrain Focus, Lateral Focus Off]
	0.92 (0.13)	[Prime Focus]
D5	1.02	[A constant in the model]
D13	-1.40 (0.18)	
D14	-1.18 (0.14)	
D15	0.75 (0.20)	
D16	-11.88 (0.07)	[Cassegrain Focus, Lateral Focus On]
	0.21	[Cassegrain Focus, Lateral Focus Off]
	-0.22 (0.13)	[Prime Focus]
D20	-0.35 (0.10)	
D21	0.77 (0.15)	

Values for D4 and D16 for Cassegrain Focus, Lateral Focus Off are from pointing measurements not reported here. Current data is insufficient for finding values for D4 and D16 under this configuration of optics.

Table 7: Rms Values Using Best-Value Coefficients

Date	Frequency (GHz)	HA rms (")	Dec rms (")
Jan 91	22	24.3	12.5
Feb 91	10	14.2	12.4
Jun 91	22	23.4	21.6
Feb 92	22	13.8	11.2
Jun 92	15	10.5	17.1
Nov 90	5	7.7	7.6
Feb 91	8	23.3	26.4
Jun 92	1.8	8.8	13.7

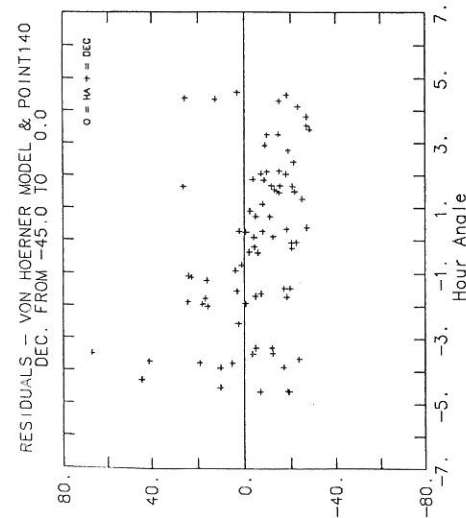
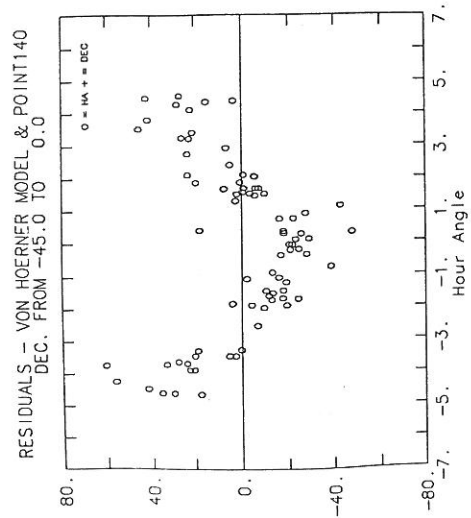
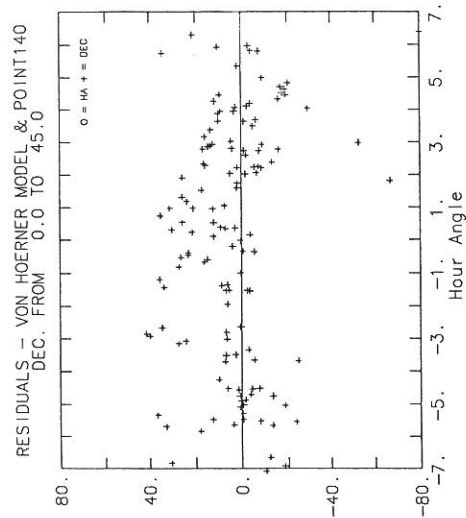
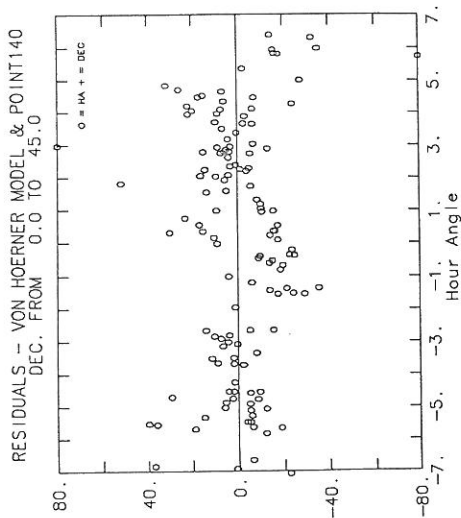
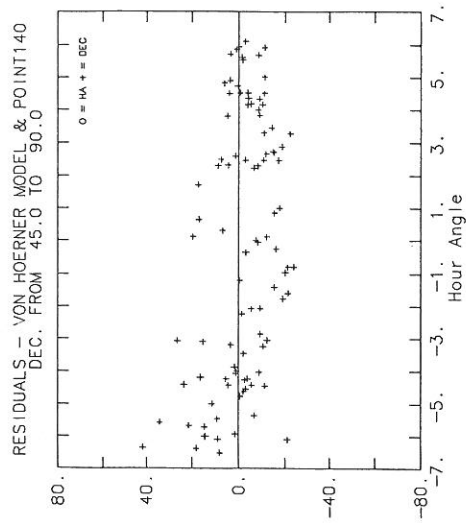
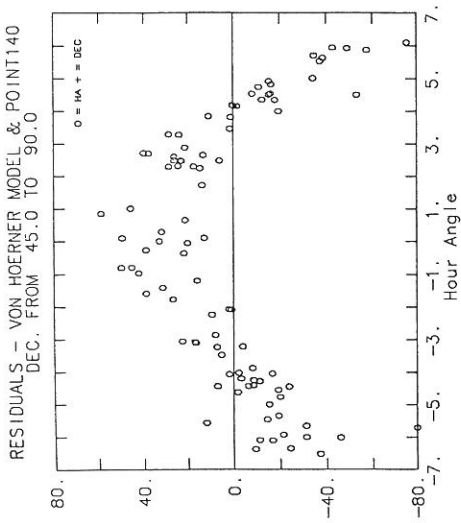


FIGURE 1

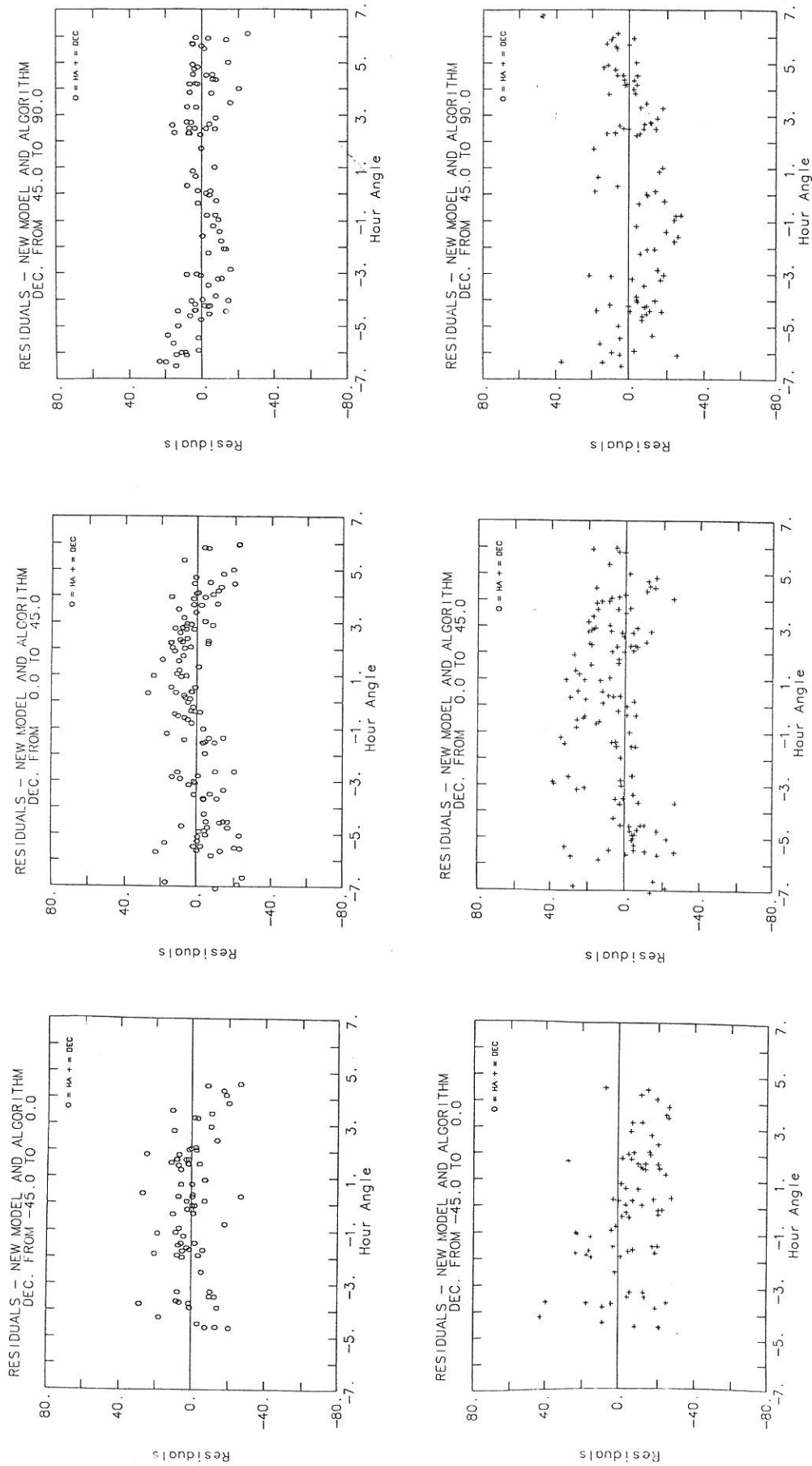


FIGURE 2

