DRAFT FOR FINAL COMMENTS

Specification of GBT Astronomical Intensity Calibration

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1. Introduction and Conventions

Owing to its offset optics, active surface, and other design features, the GBT has the potential for high accuracy amplitude or intensity calibration. Astronomical projects have a range of demands on accuracy. For many projects, 10% calibration accuracy is more than adequate, since other uncertainties in the analysis and interpretation of data are much larger than this. In other cases, calibration is quite critical, and intensity uncertainties as low as 1% are desired, at least in a relative since, and if possible, in absolute terms. GBT systems and methodologies should be designed to achieve these high accuracies whenever possible. The goal for *relative* calibration, scan to scan and across the frequency bandwidth, is thus 1%. Absolute calibration depends on the accuracy to which flux density standards are known, and is a separate issue.

There will be two, broad types of intensity scaling for GBT receivers: antenna temperature and flux density conversions. All receivers will have a noise temperature calibration device. For frequencies through 50 GHz (Q-band), receivers will have a noise diode whose output is injected into the receiver waveguide, generally between the feed and the first amplifier. Receivers above 50 GHz will have absorbing load calibrators -- chopper wheels or vanes -- that will be moved in front of the feed. Either technique will produce a calibrated system temperature (T_{sys}) array that can be used to scale the data in antenna temperature.

The second type of calibration will be the conversion to a flux density scale in Janskys. This will be done by observing a standard flux density calibrator source and determining the appropriate scale factors. As will be described below, one option is to produce a direct scaling factor (Janskys per count) that can be applied to the data, independent of the receiver calibration noise source. The more common approach will be to transfer the flux density scaling to the noise source, which can then be used for subsequent observations, and will take care of receiver gain variations that may occur after the standard calibrator is observed.

For frequencies below ~20 GHz, the flux density of standard calibrators is known to good accuracy. At higher frequencies, calibration is more problematic. For observations of extended sources, such as many Galactic spectral line observations, observers may be inclined to work in calibrated antenna temperature, rather than performing an immediate conversion to flux density.

Four choices will be provided to GBT observers for the display of intensity scaling:

- T_A' (T_A-prime: antenna temperature corrected for atmospheric attenuation)
- T_A^* (T_A-star: antenna temperature corrected for atmospheric attenuation and rear spillover)
- T_{MB} (main beam brightness temperature), and
- S_ν (flux density in Janskys).

Each choice will serve a particular need, including prevailing conventions and individual preferences, and the fact that the state of knowledge of the source brightness distributions and antenna patterns may indicate a particular scale. The data arrays in the Measurement Set (MS) will be stored in an uncalibrated format, so that the observer can choose different calibration options for offline reduction. These intensity scales are all related by conversion factors, and there will be values in the data set that will contain all the information necessary to display the data in

any of these formats, provided that valid scale factors have been supplied. Some calibration factors will be provided by the GBT staff, and others must be measured by the observers. Standard observing algorithms will be available for determining all required calibration factors.

In the initial implementation of AIPS++ data reduction, calibration will be treated on a scan-by-scan basis. Information required for calibration will be written into Measurement Set for each scan, either by the Filler or by a data reduction process. For initial reduction, calibration scaling for each scan will be based on best knowledge at the time the scan was taken (in the case of atmospheric opacity, for example), although it will be possible to re-process the data with better values after the run concludes.

In the longer term, AIPS++ calibration will be based around a Measurement Equation (ME) that provides a complete transfer function from raw to calibrated data. The ME allows observations to be treated as single data set, as opposed to a collection of individual scans. Among other things, the ME will allow variable calibration factors, such as atmospheric opacity, to be interpolated over time to achieve best, final results. The ME concept will be introduced gradually into single dish data reduction. This document specifies only the calibration formalism, not the method of implementation.

1.1 T_A' definition

T_A' (T_A-prime) is defined as the antenna temperature in units of Kelvins, corrected for atmospheric attenuation:

$$T_A' = T_A \exp(\tau_o A) \qquad [1.1.1]$$

where T_A is the conventional antenna temperature, as might be measured as an ON-OFF source difference, typically scaled from a calibrated noise source. τ_o is the atmospheric opacity at the zenith, and A is the number of airmasses relative to the zenith. When the atmosphere is uniform and plane-parallel, the conversion from zenith distance (z) or elevation angle (El) is

$$A = \sec(z) = 1/\sin(El) \qquad [1.1.2]$$

1.2 T_A^* definition

T_A^* (T_A-star) is defined as the antenna temperature in units of Kelvins, corrected for atmospheric attenuation and for rear spillover, ohmic loss, and blockage efficiency:

$$T_A^* = T_A \exp(\tau_o A)/\eta_l \qquad [1.2.1]$$

or

$$T_A^* = T_A'/\eta_l \qquad [1.2.2]$$

where η_l is the rear spillover, ohmic loss, and blockage efficiency. η_l is the fraction of antenna power that falls on the forward hemisphere, and $(1-\eta_l)$ is the fraction of antenna power that usually falls on the rear hemisphere and is usually terminated at near-ambient temperature. T_A^* is often the most general antenna temperature that can be quoted without detailed knowledge of the source brightness distribution and the antenna power pattern. It must be noted that at low elevation angles, in particular, rearward spillover may fall on the sky, and forward spillover on the ground.

The antenna temperature of blank sky has contributions from the atmosphere itself, rear spillover, scattering, and ohmic losses, the fraction of forward spillover that falls on the ground at very low elevations, and the cosmic background emission, as attenuated by the Earth's atmosphere:

$$T_A(sky) = \eta_l T_M[1-\exp(-\tau_o A)] + (1-\eta_l)T_{spill} + T_{fss}(z) + \eta_l[T_{cbr} + T_{gal}(f)]\exp(-\tau_o A), \quad [1.2.3]$$

where T_M is the mean temperature of the atmosphere, T_{spill} is the effective temperature of the rear spillover, $T_{fss}(z)$ is the forward spillover and scattering temperature, which is a function of zenith distance z, T_{cbr} is the temperature of the cosmic background radiation (~2.7K), and $T_{gal}(f)$is the non-thermal emission from the Galaxy, and is a function of frequency. Predicted spillover temperature as a function of elevation angle is illustrated in Figure 1, which was taken from the theoretical calculations of Srikanth in GBT Memo 16.

The rear spillover efficiency, η_l, can be determined by performing tipping scans (antenna temperature measurements at a variety of elevation angles) and fitting Equation [1.2.3] for τ_o and η_l. Once η_l is measured, it will be stored in the calibration files for each receiver, and automatically written to the header of each scan.

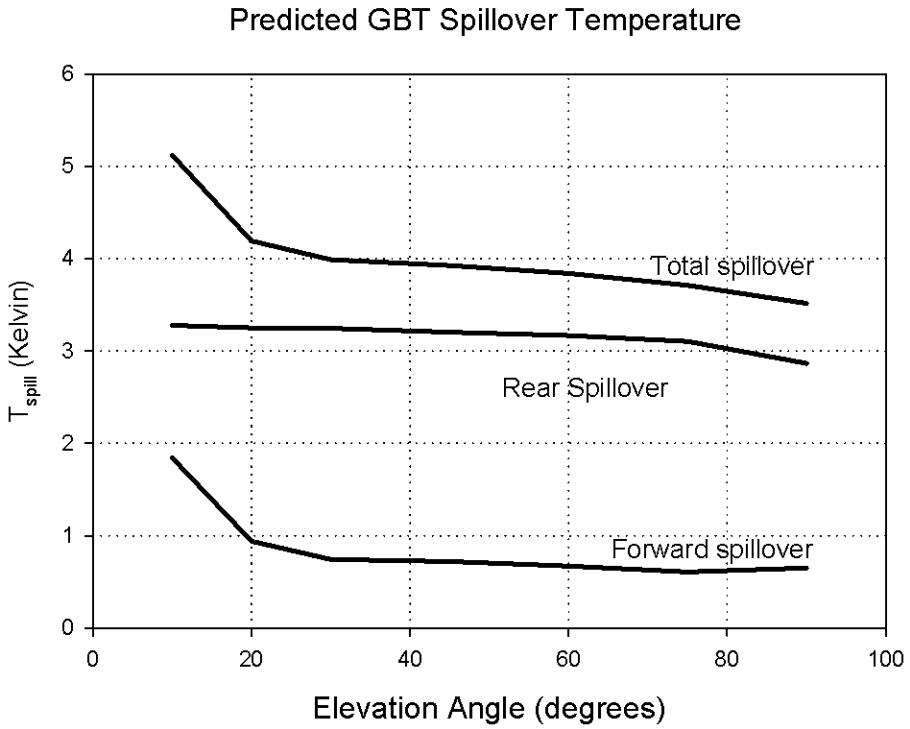

Figure 1 – Calculated GBT spillover temperatures (Srikanth; GBT Memo 16).

At higher frequencies, the temperature values, T_M, T_{spill}, T_{cbr} should be replaced with the equivalent Rayleigh-Jeans temperature defined as

$$J(T,f) = \frac{hf/k}{\exp\left[\frac{hf}{kT}\right]-1} \quad [1.2.4]$$

1.3 T_{MB} Scale

The main beam brightness temperature scale is defined as the convolution of the brightness temperature distribution of the source and the main diffraction beam of the antenna. This scale is appropriate for observation of sources that are comparable to the main beam. Formally, we can define this as

$$T_{MB} = T_A' / \eta_M \qquad [1.3.1]$$

or

$$T_{MB} = (T_A^*) \frac{\eta_l}{\eta_M} \qquad [1.3.2]$$

where η_M is the conventional beam efficiency defined in terms of antenna temperature, corrected only for atmospheric attenuation. When computing η_M from a calibration measurement, it is important that the rear spillover efficiency, η_l, is not applied twice.

1.4 Flux Density Scale

Many astronomical observations are expressed best in physically significant flux density units, e.g., Janskys (=10^{-26}Wm^{-2}Hz^{-1}). This is particularly true for emission from sources much smaller than the beam for which flux density calibration using point sources is applicable and accurate. If the flux density calibrator is extended compared to the beam (often the case for planets, for example), a correction for the beam size must be made. Likewise, corrections to the flux density calibration scale factor must be made if the target source is extended within the telescope beam. The use of a flux density scale implies that the intensities quoted, for the beam areas measured, are on an absolute scale, and could be reproduced at another observatory. If the source is extended, this requires that the object be mapped and the measurements "cleaned" to deconvolve the antenna pattern from the source brightness distribution.

If observations are first calibrated in antenna temperature, they can be converted to flux density through the equation

$$S_\nu = \frac{2kT_A'}{\eta_A A_p} \qquad [1.4.1]$$

or

$$S_\nu = \frac{2k\eta_l T_A^*}{\eta_A A_p} \qquad [1.4.2]$$

where η_A is the conventional aperture efficiency and A_p is the physical aperture of the antenna.

As described below, we will also have a method of computing a calibration factor, S_{cal}, which is analogous to T_{cal} and can be applied directly in the initial signal processing. For spectral line observations, S_{cal} will be an array. All conversions to flux density require the observation of a standard calibrator source for each observing frequency. A large number of flux density standard sources are available, such as those listed in the VLA Calibrator List.

2.1 Noise Diode Calibration

The calibration of most contemporary cm-wave observations uses a pulsed noise diode of known intensity. Unfortunately, the unprecedented wide bandwidths of the GBT Spectrometer and the narrowness of the GBT Spectrometer's channels dictates that we adopt methods for calibration that have not been traditionally used. For example, existing algorithms usually assume the bandpass is narrow enough that the noise diode intensity can be considered a constant across that bandpass. This assumption is no longer true for the bandpasses provided by the Spectrometer in most of its modes.

The following provides a methodology for calibrating data without many of our previous assumptions.

2.1.1 Determination of $T_{cal}(f)$ for "Total Power with Cal" and "Switched Power with Cal" Observations

The first step in the calibration process is to determine the intensity of the noise diode as a function of frequency, $T_{cal}(f)$. Traditionally, the receiver engineer, using hot/cold loads, measures T_{cal} over a detection bandwidth that is a few percent of the receiver's bandpass. The engineer measures T_{cal} at frequencies separated by the detection bandwidth thereby generating a table of T_{cal} across the receiver's full bandpass. Alternatively, T_{cal} values can be derived independently from astronomical observations (§2.2).

No matter how T_{cal} is derived, we require that the values used for calibration and stored with the data in the FITS files and AIPS++ MS shall be of such a quality that linear interpolation is all that will be needed to derive T_{cal} values at any frequency covered by every receiver. We envision this will require taking the raw T_{cal} values, either determined by the engineer or astronomically, excising by hand bad values, low-pass filtering or fitting a second or third order spline or high order polynomial, and producing data tables of fitted values. We also should maintain an archive of T_{cal} values, both raw and fitted.

The algorithm we will use to convert raw T_{cal} to filtered/fitted/smoothed T_{cal} cannot be stipulated at this time but will depend upon which of various algorithms produces fitted T_{cal} values that best represent the data and satisfies our linear interpolation criteria. Staff members will need to update the tables whenever new T_{cal} values are measured, something that happens only a few times a year. If we produce fitted tables in the same format as those currently produced by the engineers for holding T_{cal} values, then the M&C system would only need to pick up the new data files instead of the raw data files.

The AIPS++ calibration process, as explained below, will require T_{cal} values for every Spectrometer channel. Because of our criteria on how the tables are to be produced, AIPS++ can use simple linear interpolation to derive intermediate frequency values. The abscissa for the interpolation should be the sky frequency of the center of each channel.

[Note that there will probably never be the case in spectral line work when a channel will be wider than the bandpass over which T_{cal} is measured. However, this will occur for continuum measurements and, as discussed in the memo "Continuum Calibration" (Maddalena, Sept. 14, 2000), AIPS++ will need to produce a gain-weighted average T_{cal} values to accommodate wide bandpass continuum observations.]

2.1.2 Averaging integrations within a scan

Currently, the GBT stores the signal and reference data for beam- and frequency-switched ("Switched Power") data in the same scan or data record. For position-switched ("Total Power") data, the signal (on-source) and reference (off-source) data are stored in separate scans. Thus, unlike frequency- or beam-switched data, "Total Power" observations require accessing and processing two scans.

Scans are usually made out of multiple integrations that the user will probably not want to see separately. For each phase within a scan, AIPS++ must perform a weighted average of all the integrations.

$$V_{phase}(f) = \frac{\sum_{i=1}^{n} V_{phase_i}(f) \cdot \frac{(t_{phase_i} - t_{blanking_i})}{\left\langle V_{phase_i} \right\rangle_{all}^2}}{\sum_{i=1}^{n} \frac{(t_{phase_i} - t_{blanking_i})}{\left\langle V_{phase_i} \right\rangle_{all}^2}}. \qquad [2.1.1]$$

The algorithm handles the cases that each phase can have a different blanking time (as might be that case when we start to use a tertiary). All integrations may not have the same phase or blanking times (for example, because an observation was aborted before an integration was finished). We're also anticipating that the system temperatures can vary significantly during an observation, as can occur at high frequencies. AIPS++ should also calculate the following scalar quantities:

$$t_{phase} = \sum_{i=1}^{n} \left(t_{phase_i} - t_{blanking_i} \right)$$

$$w_{phase} = \sum_{i=1}^{n} \frac{(t_{phase_i} - t_{blanking_i})}{\left\langle V_{phase_i} \right\rangle_{all}^2}. \qquad [2.1.2]$$

$$\sigma_{phase} = \frac{K}{\sqrt{\Delta f \cdot w_{phase}}}$$

How the data are retrieved and processed depends upon the observing mode:

- **Total Power without Cal**: Data accessed via the AIPS++ UniPOPS Jr. commands called 'sig(scan.receptor) or 'ref(scan. receptor)'. The 'sig' command should produce a $V_{sig\_on}(f)$ vector and t_{sig_on}, w_{sig_on}, and σ_{sig_on} scalars, store the result in a structure named 'vsigon', and delete the 'vsigoff' structure if it exists. The 'ref' command should produce a $V_{ref\_on}(f)$ vector and t_{ref_on}, w_{ref_on}, and σ_{ref_on} scalars, store the results in a structure named 'vrefon', and delete the 'vrefoff' structure if it exists.
- **Total Power with Cal:** Data accessed via the 'sig(scan.receptor) or 'ref(scan.receptor)' commands. The 'sig' command should produce $V_{sig\_on}(f)$ and $V_{sig\_off}(f)$ vectors and t_{sig_on}, w_{sig_on}, σ_{sig_on}, t_{sig_off}, w_{sig_off}, σ_{sig_off} scalars and store the result in structures named 'vsigon' and 'vsigoff'. The 'ref' command should produce a $V_{ref\_on}(f)$ and $V_{ref\_off}(f)$ vectors and t_{ref_on}, w_{ref_on}, σ_{ref_on}, t_{ref_off}, w_{ref_on}, and σ_{ref_on} scalars and store the result in structures named 'vrefon' and 'vrefoff'.
- **Switched Power without Cal:** Data accessed via the 'get(scan.receptor) command. The 'get' command should produce $V_{sig\_on}(f)$ and $V_{ref\_on}(f)$ vectors and t_{sig_on}, w_{sig_on}, σ_{sig_on}, t_{ref_off}, w_{ref_off}, and σ_{ref_off} scalars. The result should be stored in structures named 'vsigon' and 'vrefon' while the 'vsigoff' and 'vrefoff' structures should be deleted if they exist.
- **Switched Power with Cal**: Data accessed via the 'get(scan.receptor) command. The 'get' command should produce $V_{sig\_on}(f)$, $V_{sig\_off}(f)$, $V_{ref\_on}(f)$, $V_{ref\_off}(f)$ vectors and t_{sig_on}, w_{sig_on}, σ_{sig_on}, t_{sig_off}, w_{sig_off}, σ_{sig_off}, t_{ref_on}, w_{ref_on}, σ_{ref_on}, t_{ref_off}, w_{ref_on}, and σ_{ref_on} scalars. The result should be stored in structures named 'vsigon', 'vsigoff', 'vrefon', and 'vrefoff'.

We must provide data access routines for users who will want to use non-standard calibration routines. For them, the UniPOPS Jr. command 'getir(scan.receptor,phase,integration)' should retrieve raw V(f) data for a single

phase or integration, store $V_{phase}(f)$ and t_{phase}-$t_{blanking}$ in the 'vsig' structure, and delete the 'vsigon', 'vrefon', 'vsigoff', and 'vrefoff' structures if they exist.

2.1.3 Combining "cal on" and "cal off" data in "Total Power With Cal" and "Switched Power with Cal" observations

We will need AIPS++ UniPOPS Jr. 'sig', 'ref', and 'get' commands to average the noise diode on and off data. Both 'sig' and 'get' use the vectors and scalars stored in the structures created in §2.1.2 to create:

$$\begin{aligned} V_{sig}(f) &= \frac{\left(w_{sig\_on}V_{sig\_on}(f) + w_{sig\_off}V_{sig\_off}(f)\right)}{w_{sig\_on} + w_{sig\_off}} \\ t_{sig} &= t_{sig\_on} + t_{sig\_off} \\ w_{sig} &= w_{sig\_on} + w_{sig\_off} \\ \sigma_{sig} &= \frac{K}{\sqrt{\Delta f \cdot w_{sig}}} \end{aligned} \quad . \qquad [2.1.3]$$

The weighting of the average is necessary if the user has selected the receiver's "high" noise diode. In this case, the system temperature with the diode 'on' can double or triple the system temperature thereby producing data that is 2-3 times noisier during the noise diode 'on' phases than in the 'off' phases. When using 'high cals', weighting the average will produce data with substantially lower noise. The results of this calculation should be stored in a structure named 'vsig'.

The UniPOPS Jr.'ref' and 'get' commands will need to calculate:

$$\begin{aligned} V_{ref}(f) &= \frac{\left(w_{ref\_on}V_{ref\_on}(f) + w_{ref\_off}V_{ref\_off}(f)\right)}{w_{ref\_on} + w_{ref\_off}} \\ t_{ref} &= t_{ref\_on} + t_{ref\_off} \\ w_{ref} &= w_{ref\_on} + w_{ref\_off} \\ \sigma_{ref} &= \frac{K}{\sqrt{\Delta f \cdot w_{ref}}} \end{aligned} \qquad [2.1.4]$$

the results of which can be stored in a structure called 'vref'

At this stage it is opportune to calculate system temperatures from the above vectors. If RFI is a problem, 'sig', 'ref', and 'get' must create RFI-excised versions of V_{sig}, V_{ref}, V_{sig_on}, V_{sig_off} V_{ref_on}, and V_{ref_off} using, for example, the algorithm defined in 2.A. Using these excised vectors, 'sig' and 'get' must calculate T_{sys_sig} and 'ref' and 'get' must calculate T_{sys_ref} using:

$$\begin{aligned} \left\langle T_{sys\_sig}(f)\right\rangle_N &= \left\langle \frac{T_{cal}(f)\cdot V^{rfi}_{sig}(f)}{\left(V^{rfi}_{sig\_on}(f) - V^{rfi}_{sig\_off}(f)\right)}\right\rangle_N \\ \left\langle T_{sys\_ref}(f)\right\rangle_N &= \left\langle \frac{T_{cal}(f)\cdot V^{rfi}_{ref}(f)}{\left(V^{rfi}_{ref\_on}(f) - V^{rfi}_{ref\_off}(f)\right)}\right\rangle_N \end{aligned} \quad . \qquad [2.1.5]$$

The $< >_N$ indicates boxcar averaging over N channels where N is chosen so as produce a $<T_{sys}>$ value that is accurate to a certain percentage. That is, N is determined so that $\sigma_{Tsys}/<T_{sys}>$ is less than, say, 1%. §2.B covers how 'temp' will need to calculate N. (Note than an implicit assumption in the above formulation is that during the course of an observation the change in sky frequency due to Doppler tracking is smaller than the typical frequency scale length over which T_{cal} changes.)

If RFI is not a problem, these commands should bypass the expensive RFI excision step and instead create the same quantities using:

$$\begin{aligned} \left\langle T_{sys\_sig}(f)\right\rangle_N &= \left\langle \frac{T_{cal}(f)\cdot V_{sig}(f)}{\left(V_{sig\_on}(f)-V_{sig\_off}(f)\right)}\right\rangle_N \\ \left\langle T_{sys\_ref}(f)\right\rangle_N &= \left\langle \frac{T_{cal}(f)\cdot V_{ref}(f)}{\left(V_{ref\_on}(f)-V_{ref\_off}(f)\right)}\right\rangle_N \end{aligned} \quad . \qquad [2.1.6]$$

For in-band frequency-switched data accessed via 'get', it would be wise to store away the reference and signal T_{sys} for possible later use by UniPOPS Jr.'s 'fold' command.

2.1.4 Creating Difference Spectra in "Total Power With Cal" and "Switched Power with Cal" observations

As mentioned above, the GBT stores the signal and reference data for "Switched Power" observations in the same scan. For "Total Power" observations, the signal (on-source) and reference (off-source) data are stored in separate scans. Other than that, the creation of the difference spectrum by the AIPS++ UniPOPS JR 'temp' command follows identical paths.

The difference spectrum that 'temp' calculates is:

$$T_{diff}(f) = \left\langle T_{sys\_ref}(f)\right\rangle_N \cdot \frac{V_{sig}(f)-V_{ref}(f)}{V_{ref}(f)} \qquad [2.1.7]$$

An implicit assumption in the above formulation is that the frequency dependence of the gain does not significantly change shape between the 'sig' and 'ref' data.

We have tried to choose N such that the inaccuracy with which we can measure T_{sys} is low. However, as §2.B notes, there may be times when σ_{Tsys}/T_{sys} might be significant. If we ignore these rare cases, then the T_{sys}, theoretical rms, and weights of difference spectra are:

$$\left\langle T_{sys\_diff}(f)\right\rangle_N = \sqrt{\frac{t_{ref}\left\langle T_{sys\_sig}(f)\right\rangle_N^2 + t_{sig}\left\langle T_{sys\_ref}(f)\right\rangle_N^2}{t_{ref}+t_{sig}}}$$

$$\left\langle \sigma_{diff}(f)\right\rangle_N = K\frac{\left\langle T_{sys\_diff}(f)\right\rangle_N}{\sqrt{\Delta f \cdot \frac{t_{ref}t_{sig}}{t_{ref}+t_{sig}}}} \qquad [2.1.8]$$

$$\left\langle w_{diff}(f)\right\rangle_N = \frac{1}{\left\langle \sigma_{diff}(f)\right\rangle_N^2}$$

Note that the resulting rms is calculated exactly as if the observation had an effective integration time of:

$$t_{diff} = \frac{t_{ref}t_{sig}}{t_{ref}+t_{sig}}.$$

AIPS++ should store the resulting difference data back into the 'vsig' structure.

2.1.5 Folding in-band frequency-switched spectra

In-band frequency-switched data can be 'wrapped' or 'folded' so as to provide extra signal to noise. Invoking the AIPS++ UniPOPS Jr. command 'fold' must be an optional step since in some cases the data would be better off left 'unfolded'. It involves creating a difference spectrum with the roles of signal and reference reversed, shifting the new difference spectra by the amount of the frequency switch, and averaging the shifted difference spectrum with the unshifted, usual difference spectra. For a frequency switch of Δ, the folded spectrum is:

$$T_{fold}(f) = \frac{1}{\left\langle w_{diff}(f)\right\rangle_N + \left\langle w_{diff}(f+\Delta)\right\rangle_N}\left(\begin{array}{l}\left\langle w_{diff}(f)\right\rangle_N\left\langle T_{sys\_ref}(f)\right\rangle_N\left(\frac{V_{sig}(f)-V_{ref}(f)}{V_{ref}(f)}\right)+ \\ \left\langle w_{diff}(f+\Delta)\right\rangle_N\left\langle T_{sys\_sig}(f+\Delta)\right\rangle_N\left(\frac{V_{ref}(f+\Delta)-V_{sig}(f+\Delta)}{V_{sig}(f+\Delta)}\right)\end{array}\right). \qquad [2.1.9]$$

Weighting the average will be needed if the amount of the frequency switch is greater than the frequency scale over which T_{sys} changes substantially. Note that the traditional approximation:

$$T_{fold}(f) = \left(T_{diff}(f) - T_{diff}(f+\Delta)\right)/2$$

fails when the intensity of the spectral line of interest is more than a small fraction of T_{sys}.

If the signal and reference phases are no longer available separately when one invokes the 'fold' command, then one can rewrite the above equation using just the difference spectrum exclusively.

$$T_{fold}(f)=\frac{1}{\langle w_{diff}(f)\rangle_N+\langle w_{diff}(f+\Delta)\rangle_N}\left\{\begin{array}{l}\langle w_{diff}(f)\rangle_N T_{diff}(f)+\\ \left(\frac{1}{\left(\frac{T_{diff}(f+\Delta)}{\langle T_{sys\_ref}(f+\Delta)\rangle_N}+1\right)}-1\right)\langle w_{diff}(f+\Delta)\rangle_N\langle T_{sys\_sig}(f+\Delta)\rangle_N\end{array}\right\}. \quad [2.1.10]$$

The weights of the folded data , rms, , effective integration time, and system temperature are:

$$\begin{aligned}\langle w_{rap}(f)\rangle_N &= \langle w_{diff}(f)\rangle_N + \langle w_{diff}(f+\Delta)\rangle_N \\ \langle \sigma_{rap}(f)\rangle_N &= \sqrt{\frac{1}{\langle w_{rap}(f)\rangle_N}} \\ t_{fold} &= 2t_{diff} \\ \langle T_{sys\_rap}(f)\rangle_N &= \frac{\langle \sigma_{rap}(f)\rangle_N}{K}\sqrt{\Delta f \cdot t_{fold}}\end{aligned} \quad [2.1.11]$$

The 'fold' command should use the 'vsig' structure for input values and for storing output values.

We might also want an implementation of the folding algorithm suggested by Harvey Liszt (1997, A&A Suppl., 124, 183).

2.1.5 Averaging spectra

Spectra may need to be averaged before or after difference spectra are produced, before or after in-band frequency spectra are 'folded', and before or after scaling to T_A^*, T_{MB}, or flux density (§2.1.6). It is up to the users to decide the order of processing. In any case, the default method for averaging spectra is to perform a weighted average using the inverse square of the theoretical rms. That is:

$$T_{avrg}(f)=\frac{\sum_i \langle w(f)\rangle_N T(f)}{\sum_i \langle w(f)\rangle_N}. \quad [1.1.12]$$

The weights of the folded data, rms, effective integration time, and system temperature are:

$$\left\langle w_{avrg}(f)\right\rangle_N = \sum_i \left\langle w_i(f)\right\rangle_N$$
$$\left\langle \sigma_{avrg}(f)\right\rangle_N = \sqrt{\frac{1}{\left\langle w_{avrg}(f)\right\rangle_N}}$$
$$t_{avrg} = \sum_i t \qquad [1.1.13]$$
$$\left\langle T_{sys\_avrg}(f)\right\rangle_N = \frac{\left\langle \sigma_{avrg}(f)\right\rangle_N}{K_{avrg}}\sqrt{\Delta f \cdot t_{avrg}}\,.$$

K_{avrg} is the simple average of the K for all observations going into the average.

The averaging routine should perform sanity checks on the data to ensure Δf, number of channels, … is the same for all observations going into the average.

The AIPS++ UniPOPS Jr. command 'accum' should take the 'vsig' structure, multiply by the appropriate weight, and sum the resulting vector into an 'accumscan' structure. It should also add the weight to the scalar 'accumweight'. The 'ave' UniPOPS Jr. command would take the 'accumscan' structure, divide it by 'accumweight', calculate the resulting weights, T_{sys}, t_{avrg}, and rms and store the resulting structure back into 'vsig'. The 'sclear' command would abort an on-going average by zeroing out 'accumscan' and 'accumweight'. For those users who want to specify their own weight factors, 'accum' should take an optional weight argument that, if specified, would override the default weighting given above.

2.1.6 Conversion to T_A^*, T_{MB}, and flux density

AIPS++ will need a UniPOPS Jr. commands, "ta_cal", "tmb_cal", and "flux_cal" for scaling the resulting antenna temperatures for atmospheric attenuation and gain to flux density and the T_a^* and T_{MB} temperature scales defined in §1. A way will be needed to reduce tipping observations and store a value of atmospheric opacity (τ) for later use. η_A, η_M, and η_l must be retrievable from the scan header. Since the gain will change with telescope position, frequency, and feed and the atmospheric correction factors ($e^{\tau A}$) will change with elevation, UniPOPS Jr. must be able to determine the telescope position, feed, and frequency for every scan in order to apply proper corrections factors. The "ta_cal", "tmb_cal", and "flux_cal" commands must also have an option whereby a user can override the scaling values stored with the data.

These commands would generate the following quantities:

$$T_A^*(f) = T(f)\cdot e^{\tau\cdot A(El)} / \eta_l(Az_{calib}, El_{calib}, f, feed)$$
$$T_{MB}(f) = T(f)\cdot e^{\tau\cdot A(El)} / \eta_M(Az_{calib}, El_{calib}, f, feed) \qquad [2.1.14]$$
$$S(f) = 2kT(f)\cdot e^{\tau\cdot A(El)} / \left[Area\cdot \eta_A(Az_{calib}, El_{calib}, f, feed)\right]$$

The commands must also scale T_{sys}, weights, and σ by these same factors.

2.2 Continuum Source Calibration

For some types of observing, the observer need not rely on the engineer's T_{cal} values to calibrate their data.

Instead, observers can use continuum calibration sources. In these observations, observers will be calibrating their data in units of flux density instead of antenna temperature. The continuum calibrator needs to have a well-known flux and spectral index. The derivations below take the simplifying assumption that the calibrator is essentially a point source.

If observers wish to use a continuum calibrator, then they will have a choice of two methods:

- The first method assumes that the gain of the telescope hasn't changed significantly between the calibrator and source observation. Essentially, observations are made in the " without Cal" modes.
- The second method allows for some changes in gain. Although the observations are taken in the "with Cal" modes, the engineer's T_{cal} values are ignored. Instead, the noise diode is essentially used as a secondary calibrator to boot strap the calibration from the observation of the calibrator to that of the source of interest. An implicit assumption in this method is that the frequency dependence of the noise diode changes very little with time. This method can also be used to determine relative T_{cal} values (and, if the aperture efficiency is known, absolute T_{cal}) independently of the engineer's measurements. These methods have been used before. For example, van Zee et al (1997, AJ 113, 1638) used this technique to achieve calibration accuracies of a few percent for extragalactic HI observations. And, this technique was critical in 1996 in determining the systematic errors of up to 20% in the engineer's T_{cal} for some GBT receivers.

2.2.1 Using a continuum calibrator but no noise diode

The observations consist of measurements toward the calibrator and toward the source of interest, both of which must be taken in the "without Cal" modes. The observation of the continuum calibrator must be position switched. The individual integrations of the observations are averaged as in §2.1.2 to produce V_{calib_sig} and V_{calib_ref}. From these quantities, the AIPS++ UniPOPS Jr. command "s_sys" will derive:

$$\left\langle S_{sys}(f)\right\rangle_N = S_{calib}(f)\left\langle \frac{V_{calib\_ref}(f)}{V_{calib\_sig}(f)-V_{calib\_ref}(f)}\right\rangle_N \cdot e^{-\tau\cdot A_{calib}(El)}\eta_A(Az_{calib}, El_{calib}, f, feed), \qquad [1.2.1]$$

essentially the system temperature in units of flux density. The quantities A and η_A are those applicable to the position of the telescope when the calibration observation was made. If RFI is a problem, all V's must be "RFI-excised" using something like the algorithm in §2.A. S_{sys} must be boxcar smoothed over N channels for the same reasons T_{sys} needed to be smoothed in §2.1.3. §2.C shows how to determine the number of channels to smooth over.

The observation of the source of interest can be position, frequency, or beam switched. The individual integrations of the observations are averaged as in §2.1.2 to produce V_{src_sig} and V_{src_ref}. The calibration of the data follows that in §2.1.4 - §2.1.5 but with essentially, all the "T's" (antenna temperatures and system temperatures) in the formulation converted to units of flux density. With this in mind, there's no need to explicitly rewrite these equations. However, the steps in §2.1.6 no longer apply. Instead, the results §2.1.5 need to be scaled by $\eta_A \cdot e^{\tau A}$ where η_A is a function of frequency, telescope position of the source observation, and feed and where A is a function of the elevation of the source observation. Namely:

$$S(f) = S_{diff}(f)\cdot e^{t\cdot A(El)} \Big/ \eta_A(Az_{calib}, El_{calib}, f, feed) \qquad [2.2.2]$$

The commands must also scale S_{sys}, weights, and σ by these same factors.

2.2.2 Using a continuum calibrator with noise diode as secondary calibrator

The observations consist of measurements toward the calibrator and toward the source of interest.

2.2.2.1 Calibrator observations

The observation of the continuum calibrator must be made in the "Total Power with Cal" mode. Thus, there will be measurements of the power when the telescope is on the calibrator, off the calibrator, and with the noise diode on and off. The individual integrations of the observations are averaged as in §2.1.2 and the noise diode on and off phases are combined as in §2.1.3 to produce V_{calib_sig} and V_{calib_ref}.

The noise diode intensity, in units of flux density, is given by:

$$S_{cal}(f) = S_{calib}(f) \cdot \left\langle \frac{V_{calib\_on}(f) - V_{calib\_off}(f)}{V_{calib\_sig}(f) - V_{calib\_ref}(f)} \right\rangle_N \cdot e^{-\tau A_{calib}} \eta_A(Az_{calib}, El_{calib}, f, feed) \qquad [2.2.3]$$

where V_{calib_on} and V_{calib_off}, the average of the detected powers with the noise diodes on and off, are:

$$V_{calib\_on}(f) = \frac{w_{calib\_sig\_on} V_{calib\_sig\_on}(f) + w_{calib\_ref\_on} V_{calib\_ref\_on}(f)}{w_{calib\_sig\_on} + w_{calib\_ref\_on}}$$
$$V_{calib\_off}(f) = \frac{w_{calib\_sig\_off} V_{calib\_sig\_off}(f) + w_{calib\_ref\_off} V_{calib\_ref\_off}(f)}{w_{calib\_sig\_off} + w_{calib\_ref\_off}} \qquad [2.2.4]$$

It is important to note that A and η_A are those applicable to the position of the telescope when the calibration observation was made. . If RFI is a problem, all V's must be "RFI-excised" using something like the algorithm in §2.A. In §2.1, we state that T_{cal} values change slowly with frequency. The calibration algorithm should smooth S_{cal} to a reasonable frequency resolution. The number of channels to smooth over is discussed in §2.D

Note that $S_{cal}(f) = 2kT_{cal}(f)/ \text{Area} \cdot \eta_A$ for Boltzman's constant k and an aperture with a known projected area. Thus, it is possible to determine T_{cal} from S_{cal} if η_A is known sufficiently well. If η_A is not known, one can use variations in S_{cal} to determine relative T_{cal} values.

We will need a UniPOPS Jr command, "scal" that will be called after the user invokes "sig" and "ref" that will produce an "scal_data" structure from the smoothed S_{cal} values..

2.2.2.2 Source observations

The observation of the source of interest can be position, frequency, or beam switched. To invoke the calibration, we will need a UniPOPS Jr. command that will overwrite the T_{cal} information in the source observations with S_{cal} values stored in the "scal_data" structure. After this substitution is made, the calibration of the data follows that in §2.1.2 - §2.1.5. Essentially, all the "T's" (antenna temperatures and system temperatures) in the formulation are converted to units of flux density. With this in mind, there's no need to explicitly rewrite these equations. However, the steps in §2.1.6 no longer apply. Instead, the results of the earlier calibration steps need to be scaled by $\eta_A \cdot e^{\tau A}$ where η_A is a function of frequency, telescope position of the source observation, and feed and where A is a function of the elevation of the source observation. Again:

$$S(f) = S_{diff}(f) \cdot e^{t \cdot A(El)} / \eta_A(Az_{calib}, El_{calib}, f, feed) \qquad [2.2.5]$$

The commands must also scale S_{sys}, weights, and σ by these same factors.

2.A: RFI excision in the determination of T_{sys}, S_{sys}, and S_{cal}

The calculation of T_{sys}, S_{sys}, and S_{cal} must not be contaminated too severely by RFI. One practical though not full proof method is to determine a clipping level that will determine how much a V_{phase} can be above or below the mean before it is considered an RFI spike. In this proposed procedure, the first step is to calculate the mean and rms of $V_{phase}(f)$. The clipping level will be the rms values multiplied by a factor given by the following table.

Number of channels	rms multiplier
370	3.0
2149	3.5
15787	4.0
147160	4.5

The table was derived by assuming that in a data set of, say, 370 data points with a Gaussian error distribution, on the average only one data point will have a value that deviates from the mean by 3.0 sigma. AIPS++ will then flag as bad any channel in V_{phase} that exceeds the clipping level from the mean values.

2.B: Determining an adequately smoothed T_{sys}

As shown in §2.1.4, T_{sys} is used as a scale factor in determining difference spectra. Any inaccuracy in T_{sys} results directly in a scaling error in the data. If we want two observations of the same source to produce the same intensities to some accuracy we will need to produce T_{sys} values that are at least that accurate. The determination of T_{sys} involves a calculation that is intrinsically very inaccurate if done for every channel in a spectrum. Luckily, T_{sys} tends to vary slowly with frequency so in most cases we can boxcar smooth T_{sys} to a frequency resolution such that the smoothed T_{sys} values have the prerequisite accuracy. But, how do we determine the number of channels, N, to smooth T_{sys} over?

An unsmoothed T_{sys} is determined from:

$$T_{sys}(f) = \left(\frac{T_{cal}(f) \cdot V_{off}(f)}{V_{on}(f) - V_{off}(f)} \right). \qquad [2.B.1]$$

(Note that all V's in this section refer to data that has had any significant RFI flagged or removed beforehand. It is an insignificant difference in the determination of N whether V_{on}, V_{off}, or an average of V_{on} and V_{off} is used in the numerator.)

Even though T_{cal} can be uncertain by sometime as much as 10%, I will ignore any errors in T_{cal} since such an error is essentially a scaling error that the commissioners and astronomers would eliminate through the use of astronomical calibrators. Instead, I will concentrate on the statistical fluctuations in T_{sys} due solely to the radiometer noise in the determination of V.

If I assume the errors in V are Gaussian, then one can use the theory of error propagation to determine that:

$$\frac{\sigma_{Tsys}(f)^2}{T_{sys}(f)^2} = \left(\frac{\sigma_{off}(f)^2}{V_{off}(f)^2} + \frac{\sigma_{on}(f)^2 + \sigma_{off}(f)^2}{\left(V_{on}(f) - V_{off}(f)\right)^2} \right). \qquad [2.B.2]$$

From the radiometer equation:

$$\sigma_{on}(f)^2 = \frac{V_{on}(f)^2}{\Delta f \cdot t_{on}}$$
$$\sigma_{off}(f)^2 = \frac{V_{off}(f)^2}{\Delta f \cdot t_{off}} \quad . \qquad [2.B.3]$$

Boxcar smoothing the data over N channels should reduce all of the σ^2 by 1/N. If we want the smoothed T_{sys} to have a percentage error of at most p, (for example 0.01), then:

$$\frac{1}{N \cdot \Delta f \cdot t_{on}} \left(1 + \frac{V_{on}(f)^2}{\left(V_{on}(f) - V_{off}(f)\right)^2} + \frac{V_{off}(f)^2}{\left(V_{on}(f) - V_{off}(f)\right)^2} \cdot \left(\frac{t_{on}}{t_{off}} \right) \right) \qquad [2.B.4]$$

must be less than p^2 (10^{-4} in our example). Solving for N gives:

$$N \geq \frac{1}{p^2 \cdot \Delta f \cdot t_{on}} \left(1 + \frac{V_{on}(f)^2}{\left(V_{on}(f) - V_{off}(f)\right)^2} + \frac{V_{off}(f)^2}{\left(V_{on}(f) - V_{off}(f)\right)^2} \cdot \left(\frac{t_{on}}{t_{off}} \right) \right). \qquad [2.B.5]$$

Since $V_{off} \propto T_{sys}$ and $V_{on} \propto T_{sys} + T_{cal}$, this can also be written as:

$$N \geq \frac{1}{p^2 \cdot \Delta f \cdot t_{on}} \left(1 + \frac{\left(T_{sys}(f) + T_{cal}(f)\right)^2}{T_{cal}(f)^2} + \frac{T_{sys}(f)^2}{T_{cal}(f)^2} \cdot \left(\frac{t_{on}}{t_{off}} \right) \right). \qquad [2.B.6]$$

The calculation of N need not be too accurate. For example, an underestimate of 50% in N insignificantly increases σ_{Tsys}/T_{sys} from p=0.01 to p=0.012. Thus, all of the V's and T's in equations 1.B.5 and 1.B.6 can be mean values over all channels.

$$N \geq \frac{1}{p^2 \cdot \Delta f \cdot t_{on}} \left(1 + \frac{\langle V_{on} \rangle_{all}^2}{\left(\langle V_{on} \rangle_{all} - \langle V_{off} \rangle_{all} \right)^2} + \frac{\langle V_{off} \rangle_{all}^2}{\left(\langle V_{on} \rangle_{all} - \langle V_{off} \rangle_{all} \right)^2} \cdot \left(\frac{t_{on}}{t_{off}} \right) \right)$$
$$\geq \frac{1}{p^2 \cdot \Delta f \cdot t_{on}} \left(1 + \frac{\left(\langle T_{sys} \rangle_{all} + \langle T_{cal} \rangle_{all} \right)^2}{\langle T_{cal} \rangle_{all}^2} + \frac{\langle T_{sys} \rangle_{all}^2}{\langle T_{cal} \rangle_{all}^2} \cdot \left(\frac{t_{on}}{t_{off}} \right) \right) \qquad [2.B.7]$$

For back-of-the-envelope calculations, one can assume p=0.01, $t_{on}=t_{off}=0.5t_{dur}$, and typical low-diode values of $T_{cal}=0.1T_{sys}$. With these assumptions, $N > 4.4 \cdot 10^6/(\Delta f \cdot t_{dur})$. Once N is known, AIPS++ can determine the boxcar-smoothed value of T_{sys} as formulated in §2.1.3.

There are a few caveats about N that need to be discussed:

- For observations with long integrations or for high values of σ_{Tsys}/T_{sys}, it might be possible that N is less than one channel. In this case N can be safely taken to be equal to 1 and boxcar smoothing can be eliminated. Here, σ_{Tsys}/T_{sys} will end up being better than that requested.
- In short integration observations or for low values of σ_{Tsys}/T_{sys}, N could be larger than the number of channels available and AIPS++ will need to 'smooth' across the whole bandpass. In practice, boxcar

smoothing across the full bandpass is equivalent to using the mean of T_{sys}. (Note that this later situation is what traditionally has been used for many cm-wave telescopes.) The resulting σ_{Tsys}/T_{sys} may be significantly larger than that desired.

- With the wide bandpass of the GBT spectrometer, T_{cal} can vary significantly across the bandpass and, in some cases, T_{cal} may vary significantly over a frequency range smaller than N. If so, it is probably wise to reduce N to the frequency scale over which T_{cal} changes but which unfortunately has a side affect of increasing the inaccuracy of T_{sys}.

For example, the GBT Spectrometer in the 12.5 MHz bandwidth, 9-level mode, with two I.F. samplers for dual polarizations, produces 8192 channels each with Δf =1.5 kHz. t_{dur} needs to be 48 minutes if one wants to calculate T_{sys} for every channel! For most galactic HI work, observers traditionally would prefer a t_{dur} of 10 sec requiring that T_{sys} be calculated over ~0.4 MHz. (Luckily, T_{cal} for the 1-2 GHz receiver does not varies significantly over .4 MHz.) Thus, T_{sys} must be determined by smoothing over about 300 channels.

2.C: Determining an adequately smoothed S_{sys}

As shown in §2.2.1, S_{sys} is used as a scale factor in determining difference spectra so any inaccuracy in S_{sys} results directly in a scaling error in the data. An unsmoothed S_{sys} is determined from:

$$S_{sys}(f) = \left(\frac{S_{calib}(f) \cdot V_{calib\_ref}(f)}{V_{calib\_sig}(f) - V_{calib\_ref}(f)} \right). \qquad [2.C.1]$$

and, if we ignore all errors in S_{calib},

$$\frac{\sigma^2_{Ssys}}{S^2_{sys}} = \left(\frac{\sigma_{calib\_ref}(f)^2}{V_{calib\_ref}(f)^2} + \frac{\sigma_{calib\_sig}(f)^2 + \sigma_{calib\_ref}(f)^2}{\left(V_{calib\_sig}(f) - V_{calib\_ref}(f)\right)^2} \right) \qquad [2.C.2]$$

(Note that all V's in this section refer to data that has had any significant RFI flagged or removed.) As in §2.B, we want to know the number of channels to smooth 2.C.1 to so as to produce a value for σ_{Ssys}/S_{sys} that is less than some criteria p. Using the same methodology as in §2.B,

$$\begin{aligned} N &\geq \frac{1}{p^2 \cdot \Delta f \cdot t_{calib\_sig}} \left(1 + \frac{\langle V_{calib\_sig} \rangle^2_{all}}{\left(\langle V_{calib\_sig} \rangle_{all} - \langle V_{calib\_ref} \rangle_{all} \right)^2} + \frac{\langle V_{calib\_ref} \rangle^2_{all}}{\left(\langle V_{calib\_sig} \rangle_{all} - \langle V_{calibr\_ref} \rangle_{all} \right)^2} \cdot \left(\frac{t_{calib\_sig}}{t_{calib\_ref}} \right) \right) \\ &\geq \frac{1}{p^2 \cdot \Delta f \cdot t_{calib\_sig}} \left(1 + \frac{\left(\langle S_{sys} \rangle_{all} + \langle S_{calib} \rangle_{all} \right)^2}{\langle S_{calib} \rangle^2_{all}} + \frac{\langle S_{sys} \rangle^2_{all}}{\langle S_{calib} \rangle^2_{all}} \cdot \left(\frac{t_{calib\_sig}}{t_{calib\_ref}} \right) \right) \end{aligned} \qquad [2.C.3]$$

If p=0.01, $t_{calib\_sig}=t_{calib\_ref}$, and if a calibration source has an intensity ~10% of the system temperature, N > $2.2 \cdot 10^6/(\Delta f \cdot t_{sig})$.

There are a few caveats about N that need to be discussed:

- For observations with long integrations or for high values of σ_{Ssys}/S_{sys}, it might be possible that N is less than one channel. In this case N can be safely taken to be equal to 1 and boxcar smoothing can be eliminated. Here, σ_{Ssys}/S_{sys} will end up being better than that requested.
- In short integration observations or for low values of σ_{Ssys}/S_{sys}, N could be larger than the number of

channels available and AIPS++ will need to 'smooth' across the whole bandpass. The resulting σ_{Ssys}/S_{sys} may be significantly larger than that desired.

- With the wide bandpass of the GBT spectrometer, S_{sys} can vary significantly across the bandpass and, in some cases, S_{sys} may vary significantly over a frequency range smaller than N. If so, it is probably wise to reduce N to the frequency scale over which S_{sys} changes but which unfortunately has a side affect of increasing the inaccuracy of S_{sys}.

2.D: Determining an adequately smoothed S_{cal}

We would like S_{cal} in §2.2.2 to have a very low statistical error but unfortunately a full derivation of the statistical errors is pretty messy. If we assume $t_{sig} = t_{ref}$ and $t_{ref\_on} = t_{ref\_off} = t_{sig\_on} = t_{sig\_off} = 0.25\ t_{dur}$, and following the logic outlined in §2.B, then:

$$\frac{\sigma^2_{Scal}(f)}{S^2_{cal}(f)} = \frac{2}{\Delta f \cdot t_{dur}} \left(\frac{V_{calib\_on}(f)^2 + V_{calib\_off}(f)^2}{\left(V_{calib\_on}(f) - V_{calib\_off}(f)\right)^2} + \frac{V_{calib\_sig}(f)^2 + V_{calib\_ref}(f)^2}{\left(V_{calib\_sig}(f) - V_{calib\_ref}(f)\right)^2} \right) \qquad [2.D.1]$$

is an adequate approximation to the fractional statistical error in S_{cal}. (Note that all V's in this section refer to data that has had any significant RFI flagged or removed beforehand.)

As in §2.B, we need to determine the number of channels to boxcar smooth S_{cal} over to produce a $\sigma_{Scal}/S_{cal} < p$ where p is something like 0.005 or less. Using the ideas in 2.B:

$$N \geq \frac{2}{p^2 \cdot \Delta f \cdot t_{dur}} \left(1 + \frac{\left\langle V_{calib\_on} \right\rangle^2_{all} + \left\langle V_{calib\_off} \right\rangle^2_{all}}{\left(\left\langle V_{calib\_on} \right\rangle_{all} - \left\langle V_{calib\_off} \right\rangle_{all} \right)^2} + \frac{\left\langle V_{calib\_sig} \right\rangle^2 + \left\langle V_{calib\_reff} \right\rangle^2}{\left(\left\langle V_{calib\_sig} \right\rangle_{all} - \left\langle V_{calib\_reff} \right\rangle_{all} \right)^2} \right). \qquad [2.D.2]$$

For example, if p = 0.005, if the intensity of the calibrator is about that of the noise diode, and if the noise diode is 10% of the system temperature, then $N > 38.8 \cdot 10^6/(\Delta f \cdot t_{dur})$.

As in 2.B, there are a few caveats:

- For observations with long integrations or for high values of σ_{Scal}/S_{cal}, it might be possible that N is less than one channel. In this case N can be safely taken to be equal to 1 and boxcar smoothing can be eliminated. Here, σ_{Scal}/S_{cal} will end up being better than that requested.
- In short integration observations or for low values of σ_{Scal}/S_{cal}, N could be larger than the number of channels available and AIPS++ will need to 'smooth' across the whole bandpass. The resulting σ_{Scal}/S_{cal} may be significantly larger than that desired.
- With the wide bandpass of the GBT spectrometer, S_{cal} can vary significantly across the bandpass and, in some cases, S_{cal} may vary significantly over a frequency range smaller than N. If so, it is probably wise to reduce N to the frequency scale over which S_{cal} changes but which unfortunately has a side affect of increasing the inaccuracy of S_{cal}.

Definitions

V_{sig}, V_{ref}	Power or counts in arbitrary units in the 'signal' (on source or on frequency) phase or reference (off source or off frequency) phase.
V_{on}, V_{off}	Power with the noise diode on or off.
V_{calib}, V_{src}	Power on a continuum calibrator or source of interest.
V_{phase_I}	Power in a integration within a scan for a particular phase (noise on or off, signal or

	reference).
V^{rfi}	RFI-excised power or counts.
$< >_N$	A quantity boxcar smoothed over N channels.
$< >_{all}$	A quantity averaged over all channels.
t_{phase}	The time over which data were collected for a particular phase (in seconds).
$t_{blanking}$	Blanking time at the start of a phase
t_{dur}	The duration of a scan corrected for blanking.
F	Frequency in Hz.
Δf	Channel width.
w_{phase}	Weights for a particular phase.
σ_{phase}	The rms or standard deviation for a particular phase in units of either V or T.
$T_{cal}(f)$	Noise diode values in Kelvin as provided by the engineer.
$S_{calib}(f)$	Flux density of a continuum calibrator.
$S_{cal}(f)$, $S_{sys}(f)$	Noise diode and system 'temperatures' in Jy as determined by observations of a continuum calibrator.
$T_{sys}(f)$	System temperature in units of T_{cal}.
T(f)	Spectrum in units of T_{cal}.
Δ	Change in frequency for frequency-switched observations.
K	Quantization and channel separation factor in the radiometer equation for the particular backend and mode of observing.
η_A, η_M, η_l	Aperture, beam, and rear splillover efficiency
τ, A	Atmospheric opacity and number of atmospheres (~1/sin(elevation)
Area	Projected geometric area of antenna
K	Boltzman's constant

Any quantity that has an explicit frequency argument (e.g., V(f)) indicates a vector, all others are scalar quantities.

2.3 Absorbing load (chopper wheel) calibration

Absorbing load calibration, often known as chopper wheel calibration, requires different formalism and methodology than noise diode calibration. The primary reason for this is that when the absorber is over the feed, it completely blocks the sky emission. This has some benefits in that it provides an automatic way to correct for atmospheric attenuation. The computation of the calibration scale factor, T_{cal}, for absorbing load calibration is rather involved. Since the GBT will not have any receivers using this calibration method for a year or more, we will defer specification of this type of calibration to a later date.

3. Antenna, Optics, and Atmospheric Gain Corrections

To achieve the desired level of relative, as well as absolute, calibration, a number of loss factors must be included:

- Atmospheric attenuation losses
- Antenna efficiency terms
- Gain-Elevation losses, caused by antenna surface distortions as a function of elevation angle
- Feed offset losses, produced when feeds are displaced from the optical axis, or have non-uniform performance.

It should be noted that one needs knowledge of many of these terms, with the possible exception of the antenna efficiency terms, even if cross scaling against a flux density standard is used. This is the case because many of these terms are functions of sky position and will vary between the calibrator position and the target source position.

3.1 Atmospheric Attenuation Correction

For high accuracy calibration, atmospheric attenuation corrections should be made at almost all GBT observing frequencies. This is particularly true of observations at 10 GHz or higher, but the effects of the atmosphere can be seen at frequencies as low as 1.4 GHz. If observing targets are at low elevation angle, then corrections are even more important: an elevation angle of 30° corresponds to 2 airmasses, i.e., sec(z)=2; an elevation of 19.5° corresponds to 3 airmasses.

The atmospheric opacity can be determined by tipping observations and fitting Equation [1.2.3] for τ_o, using an AIPS++ routine, to be provided. The routine will write τ_o to the MS, where it will be available for scaling each scan.

3.2 Efficiency corrections

The rear spillover efficiency and various other aperture, beam, and spillover efficiencies will be measured by the GBT staff and stored in the receiver calibration tables. These will be written to the MS for each scan. Any updates to these factors must be done be editing the MS and re-reducing the data.

3.3 Gain-Elevation Effects

The GBT will have a substantial gain-elevation curve in Phase I operation, before the active surface is in use. The predicted gain-elevation curves for 8 GHz and 20 GHz are shown in Figure 2. The GBT staff will have the responsibility of measuring gain-elevation curves for each receiver band. These will be tabulated, and must be read and applied by AIPS++. In Phase II, when the active surface is in open loop mode, the gain curve will be much flatter, but will still be present. In Phase III, with closed loop metrology and active surface, the gain may be constant with elevation angle, but this will have to be confirmed.

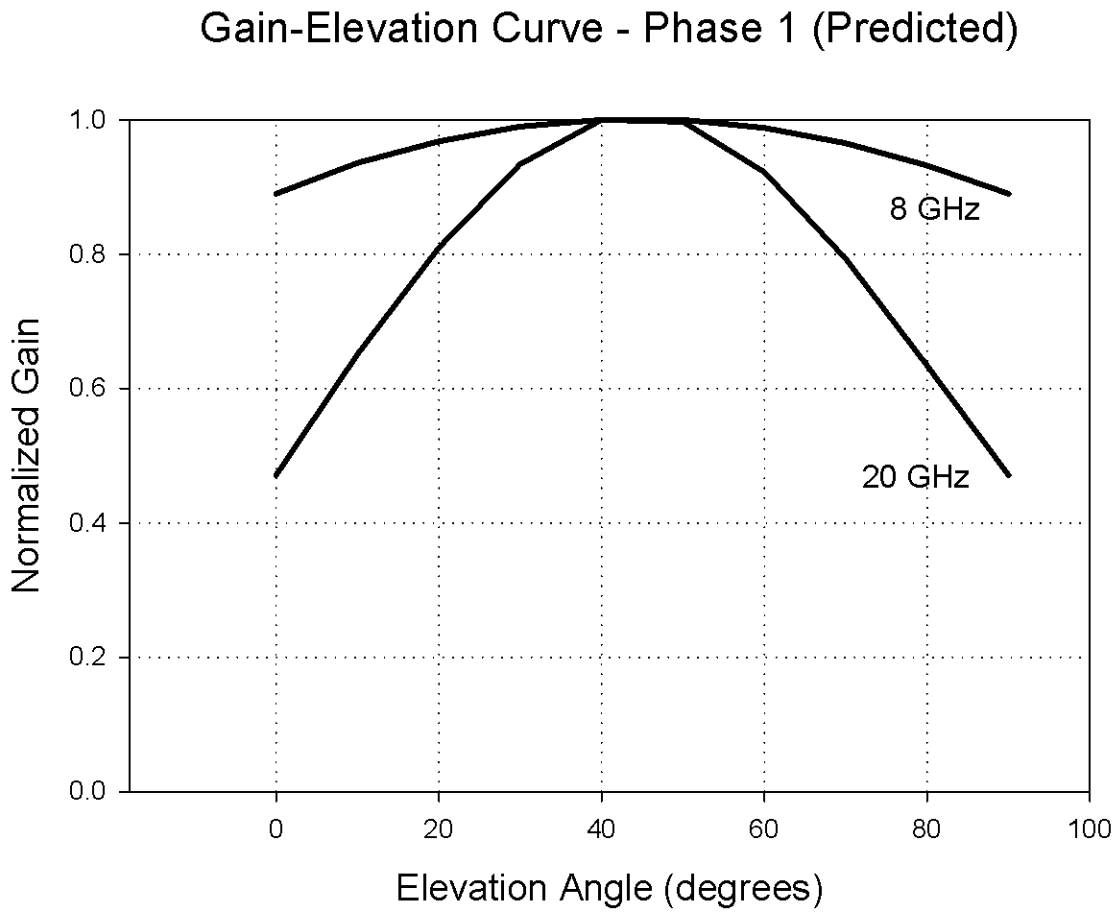

Figure 2 – Predicted normalized gain-elevation curves at 8 GHz and 20 GHz.

3.4 Feed offset gain corrections

As feeds are moved off the optical axis or boresight, the gain will vary owing to aberration losses. Furthermore, feed responses may vary simply from manufacturing differences. In general, a lookup table is needed that contains the relative gains of the different feeds in a receiver. The response of the various feeds must be relative to the efficiency factors for that receiver.

3.5 Expression for T_{MB}

Given the effects discussed above, the final expression the main beam brightness temperature is given by

$$T_{MB} = \frac{T_A \exp(\tau_o A)}{\eta_M g_M(az, el, f, feed)} \qquad [3.5.1]$$

where η_M is the conventional beam efficiency at the peak of the gain-elevation curve, and g_M(az,el,f,feed) is the normalized beam efficiency versus position curve, which is a function of elevation (and azimuth for some telescopes), frequency, and receiver feed. g_M will typically be provided by a lookup table.

It should be emphasized that T_{MB} is appropriate for an object that is comparable in size to the main diffraction beam, and the concept of beam efficiency scale factors, $\eta_M g_M$, are only valid in this case. If the angular size of the object is much smaller than the main beam, then it is preferable to express intensity directly in flux density units using the aperture efficiency (see below). If the angular size is larger than the main beam, antenna sidelobes may couple to the source. In this case, one should use T_A' or T_A*, if detailed knowledge of the source and beam distributions are lacking. The preferred approach, in all cases, is to deconvolve the beam from the source, but that requires detailed knowledge of the beam and the large-scale intensity distribution of the source.

3.6 Expression for S_ν

Similarly to T_{MB}, apparent flux density measurements must be corrected for atmospheric attenuation, position-variable aperture efficiency, and the relative efficiency of the different feeds in a multi-feed receiver:

$$S_\nu = \left[\frac{2kT_A}{A_p \eta_A}\right] \frac{\exp(\tau_o A)}{g_A(az, el, f, feed)} \qquad [3.6.1]$$

where η_A is the aperture efficiency at the peak of the gain-elevation curve, g_A(az, el, f, feed) is the normalized gain-elevation curve as a function of elevation (and azimuth, for some telescopes), frequency, and feed in a multi-feed receiver. This equation can be used for offline computation of flux density if the observation is calibrated in units of antenna temperature. In some cases, particularly at low frequencies where the atmosphere, antenna, and receiver gains are quite constant, observers may wish to determine a direct scaling factor against a continuum source, which obviates the need of a secondary noise source. As described in Appendix A, the scaling relationship is given by

$$S_{sys} \propto S_{continSrc} \frac{V_{continOff}}{V_{continOn} - V_{continOff}} \qquad [3.6.2]$$

where $S_{continSrc}$ is the flux density of the calibrator, $V_{continOn}$ are the measured response of the source in arbitrary units, and $V_{continOff}$ is the arbitrary response at a nearby off position.

The more common approach will be to determine an equivalent flux density of the calibration noise source, given by (Appendix A)

$$S_{cal} \propto S_{continSrc} \frac{V_{calOn} - V_{calOff}}{V_{continOn} - V_{continOff}} \qquad [3.6.3]$$

where V_{calOn} and V_{calOff} are the system responses when the noise source is on and off, respectively. Proper application of the S_{cal} method requires that atmospheric attenuation and variable gain factors be applied. If an apparent (uncorrected) flux density, S_{app}, is measured using the S_{cal} method, the true flux density of the target source will be

$$S_\nu = \frac{S_{app} \exp(\tau_o A)}{g_A(az, el, f, feed)}, \qquad [3.6.4]$$

where

$$S_{app} = \frac{V_{On} - V_{Off}}{V_{calOn} - V_{calOff}} S_{cal}, \qquad [3.6.5]$$

and where

$$S_{cal} = S_{continSrc} \frac{V_{calOn} - V_{calOff}}{V_{continSrcOn} - V_{continSrcOff}} \exp(-\tau_o A) g_A(az, el, f, feed), \qquad [3.6.6]$$

V_{On} and V_{Off} could refer to either a line or continuum measurement. Note that when one measures a "Janskys/Kelvin" scaling factor by observing a calibrator source, the atmospheric attenuation and gain-elevation and feed losses will be implicitly corrected. These factors must be removed from S_{cal} or subsequent flux density values will be overscaled – see Equation [3.6.1]. Thus, before determining S_{cal}, the observer should first determine the atmospheric opacity and ensure that the gain tables are being applied.

Joe's section

Appendix 1
Calibration Strategies and Philosophy

R.Fisher

A1.1 Intensity Calibration

The primary objective of the flux density calibration observations and data reduction procedures is to transfer the flux density scale measured with one or more continuum radio sources to the observed spectral line intensity. There are a number of ways to do this, each with its own advantages and disadvantages, as described below. Measurements of a receiver's calibration noise source temperatures in the laboratory are useful for determining approximate system temperatures and aperture efficiencies, but these measurements may not be appropriate for accurate spectrum calibration because of the fine-scale frequency dependence of the cal values and the unknown, frequency-dependent differences in coupling coefficients of the feed and noise source to the first amplifier. Observing a continuum radio source calibrator with the same signal path as is used to observe the spectral line circumvents a number of systematic errors present in less direct methods. This is possible with 3- and 9-level samplers because the autocorrelation intensity information is retained in distorted form, and the quantization correction removes this distortion.

If we give up the assumption that the receiver's noise source intensity and the system temperature are constant across the spectrometer passband, we must carefully assess the added random noise associated with channel-by-channel calibration. An observer may want to choose a data reduction scheme that is most appropriate to the specific conditions of the observing task.

There is a long-running debate about whether the receiver's calibration noise source should be fired intermittently during all scans or fired only in separate scans that are taken for the sole purpose of measuring the cal. Firing the cal during an astronomical observation adds to the system temperature, but the same integration time can be used for the cal and radio source measurement, and the gain and system temperature of the receiver is monitored during the time of the observation rather than a short time before or after. In this age of stable receivers, these differences are subtle ones. With adequate isolation, switching the cal on and off should have little or no affect on the transfer characteristics of the receiver, but there was at least one instance in a 140-ft observation where this did happen. The GBT receivers should be checked for affects like this. Since the calibration sequence is largely a matter of observer choice, both methods should be accommodated with standard observing procedures and data reduction tasks. To help a new observer choose between the two methods a couple of example calculations of the signal-to-noise-ratio trade-offs using typical cal and system temperature values, integration times, and scan change overheads should be provided along with a brief description of the advantages and disadvantages of each method. An "SNR calculator" might be provided for both methods that allows an observer to plug in his or her own parameters.

A1.2 Constant T_{cal} Measured in the Lab

With the old two-level (one-bit) autocorrelators we were forced to assume that the receiver's internal noise source intensity and the system temperature were both constant across the spectrometer passband. The T_cal value was measured in the lab, and T_{sys} was measured at the time of observation with a separate square-law detector connected to the output of the same filter/amplifier that drove the one-bit sampler. Most spectra were displayed in antenna temperature units and, when appropriate, converted to flux density with a separate antenna efficiency calibration. This same scheme has been used with three-level correlators, partly for historical reasons, and the underlying assumptions were and are good enough for many purposes.

From the total power detectors the system temperature is computed from

$$T_{sys} = 0.5 T_{cal} \frac{TP_{calOn} + TP_{calOff}}{TP_{calOn} - T_{PcalOff}} \quad . \qquad [A.1]$$

The calibrated difference spectrum is then

$$T_A = T_{sys} \frac{V_{onSource} - V_{offSource}}{V_{offSource}}, \qquad [A.2]$$

where the quantities in parentheses are normalized autocorrelator spectra. If the receiver's noise source is fired during the observation to get the instantaneous system temperature, the onSource and offSource spectra are sums of the spectra measured with the cal on and off.

With a 3- or 9-level correlator the total power detectors are not required, and the system temperature may be computed for each channel in the passband.

$$T_{sys} = 0.5 T_{cal} \frac{V_{calOn} + V_{calOff}}{V_{calOn} - V_{calOff}}, \qquad [A.3]$$

where calOn and calOff are usually from the offSource position. Now T_{sys} is a spectrum, but it is a noisy quantity because of dividing by the noisy spectrum, (calOn - calOff). If

$$(\text{onSource} - \text{offSource}) << (\text{calOn} - \text{calOff}) \qquad [A.4]$$

everywhere in the spectra, dividing by a noisy T_{sys} causes little harm, but this condition is not always met, particularly if the spectral line is moderately strong or there is significant continuum radiation from the spectral line source. The noise in T_{sys} can be reduced while retaining most of its measured frequency dependence by fitting a low-order analytic function to it with emphasis on the spectral range of interest. Alternatively, the T_{sys} spectrum could be smoothed with a convolving function, but this leaves residuals at places in the spectrum where the second derivative of T_{sys} as a function of frequency is high. Also, a convolving function leaves correlated noise on scales shorter than the convolving width. In either case, RFI needs to be removed before a final fit or convolution is derived. Errors in the fit or convolution will produce second-order distortions to the final S_{line} spectrum, but these may be of less concern than the added random noise. Inspection of the intermediate spectra produced in the calibration process is required to optimize the procedure in difficult cases.

A1.3 System Noise Equivalent Flux Density Calibration

If the receiver's internal noise source intensity is not constant with frequency and cannot be measured with sufficient accuracy and frequency density, as is the case with the GBT receivers and wide spectrometer and spectral processor bandwidths, it will often be best to bypass the system temperature calculation for astronomical calibration. One way to do this is to calibrate the system noise power spectrum in units of flux density by measuring its ratio to the noise power from a continuum radio source calibrator.

$$S_{sys} = \frac{S_{continSrc} \bullet V_{continOff}}{V_{continOn} - V_{continOff}}, \qquad [A.5]$$

where all quantities are functions of frequency. $S_{continSrc}$ is the continuum source flux density; continOn and continOff are the spectra measured on and off the radio source, respectively; and S_{sys} is the system noise power in flux density units. Then the calibrated spectral line difference spectrum is

$$S_{line} = \frac{S_{sys} \bullet (V_{onSource} - V_{offSource})}{V_{offSource}} . \qquad [A.6]$$

In this case the criterion for not adding random noise in a channel-by-channel calibration is

$$(\text{onSource} - \text{offSource}) << (\text{continOn} - \text{continOff}) \qquad [A.7]$$

everywhere in the spectra, which is bit better than inequality [A.4] since a moderately strong continuum source can be chosen. Keep in mind that the conditions given by inequality [A.4] or [A.7] must be scaled by the square root of the ratio of total integration times used in the measurements of quantities in equations [A.2] and [A..3] or [A.5] and [A.6], respectively. The same trick of fitting an analytic function or smoothing may be applied to S_{sys} as was suggested for T_{sys} in the previous section to reduce the added noise of the calibration.

A side benefit of the calibration scheme specified by equations [A.5] and [A.6] is that it has the potential for removing standing wave ripple on the continuum component of the spectral line radio source because the frequency dependent gain of the whole system, not just the receiver, is accounted for. How well this benefit can be exploited depends on a number of factors that need to be investigated, such as how closely the spatial distribution of the calibrator continuum source in the telescope beam must match the distribution of the spectral line source continuum radiation.

A potential drawback of this calibration scheme is that it assumes that the system noise power spectrum and the (frequency-dependent?) antenna gain are the same in the continuum calibrator 'off' measurement as it is in the spectral line 'off' scan. If the calibrator must be measured on a different azimuth sky track and, particularly a different elevation sky track than the spectral line 'off' scan(s), the stability of these system parameters will be compromised to some extent. With little continuum radiation from the spectral line source, this is usually not a big concern.

A1.4 Noise Source Flux Density Calibration

The potential problem of system noise spectrum instability mentioned in the last section may be circumvented by using receiver's internal noise source instead of the system noise to carry the flux density scale to the spectral line measurement. The receiver's noise cal will generally not be influenced by telescope position, and it is likely to be more stable with time than the receiver's gain. This calibration transfer relies only on the stability of the noise source intensity and of the transfer function of the cal and sky signals into the first amplifier. Both of these are good assumptions to first order, but there may be second-order changes in the sky-to-amplifier transfer function beyond the normal elevation dependence of the antenna gain.

In this scheme the noise source calibration calculation is

$$S_{cal} = \frac{S_{continSrc} \bullet (V_{calOn} - V_{calOff})}{V_{continOn} - V_{continOff}} , \qquad [A.8]$$

where the quantities to the right of the equal sign are as described above, and S_{cal} is the noise source spectrum in flux density units. Transfer of this flux density scale to the spectral line measurement is then

$$S_{line} = S_{cal} \frac{V_{lineOn} - V_{lineOff}}{V_{calOn} - V_{calOff}} , \qquad [A.9]$$

where new calOn and calOff measurements are done for the spectral line observations; lineOn and lineOff are spectral line object 'on' and 'off' spectra; and S_{line} is the difference spectrum in flux density units. Again, all quantities are functions of frequency. Equations [A.8] and [A.9] work well only if

$$S_{line} << S_{cal} \qquad [A.10]$$

and

$$S_{line} << S_{contInSrc} \qquad [A.11]$$

(assuming roughly equal integration times), where S_{line} includes any continuum flux, otherwise the random noise in the calibrated S_{line} spectrum will be greater than in the uncalibrated difference spectrum because of dividing by noisy cal on-off and continuum source on-off differences. The extra random noise introduced by the calibration transfer may be reduced by an analytic fit to or smoothing the S_{cal} spectrum, the quantity (V_{calOn} - V_{calOff}), or the ratio of the two depending on the relative intensities of the three quantities in the inequalities [A.10] and [A.11]. The same caveats about RFI removal and errors in the analytic function fit or convolution apply here as they did in the two previous sections. Again, inspection of the intermediate spectra produced in the calibration process may be required to optimize the procedure in difficult cases.

In the equations [A.2], [A.3], [A.8], and [A.9] we have hidden the fact that there may be two calOn's (off-source-cal-on and on-source-cal-on), two calOff's, two continOn measurements, etc., if the cal is fired during the continuum and spectral line measurement scans. In some cases, two components are measured with different system temperatures and should be weighted accordingly in the process of combining them. For example, the (V_{calOn} - V_{calOff}) difference when pointed toward the continuum source will be at a higher system temperature than when looking at the reference sky position. A good approximation to the relative weights is the square of the total powers when the two differences are measured, averaged over the spectrum.

$$W = \left\langle \frac{V_{offSource\_calOff} + V_{offSource\_calOn}}{V_{onSource-calOff} + V_{onSource-calOn}} \right\rangle^2 , \qquad [A.12]$$

where $\langle\ \rangle$ denotes the mean; if RFI is a problem in the spectrum, a median might be better than the mean. Then

$$V_{calOn} - V_{calOff} = \frac{W((V_{onSource-calOn} - V_{onSourcc-calOff}) + (V_{offSource-calOn} - V_{offSource-calOff}))}{W+1} . \qquad [A.13]$$

If the quantization-corrected spectra are perfectly linear in their power measurement, then (on-source-cal-on - on-source-cal-off) should be identical to (off-source-cal-on - off-source-cal-off) to within the noise. Because of the random noise this is not a terribly stringent test of linearity, but it is a sanity check that should be part off

commissioning. Inspection and understanding of other intermediate spectra and differences in the data reduction process should also be accomplished in system debugging, and a list of requested displays is listed in the next section.

A1.5 Displayed Quantities

Most observers will need a fairly limited number of data displays and prefer a reasonably automated system of data analysis with the assumption that most data operations have been thoroughly debugged. However, for these tests and continuing GBT commissioning a variety of data displays are required to inspect each step of the data flow.

on-source-cal-on
on-source-cal-off
off-source-cal-on
off-source-cal-off
(on-source-cal-on - on-source-cal-off)
(off-source-cal-on - off-source-cal-off)
(on-source-cal-on - off-source-cal-on)
(on-source-cal-off - off-source-cal-off)
contin On - contin Off
cal On - cal Off
$S_{contin Src}$
$S_{cal} = S_{contin Src}$ * (cal On - cal Off) / (contin On - contin Off) line On - line Off
$S_{line} = S_{cal}$ * (line On - line Off) / (cal On - cal Off) T_sys = T_cal * cal Off / (cal On - cal Off)