Theoretical Ratio of Beam Efficiency to Aperture Efficiency

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A number of calibration projects have asked for an easy way to determine the beam efficiency. I have found it easiest to convey this as a ratio of the beam efficiency to the better-known aperture efficiency. As I will show, the standard pointing observations obtained during every GBT project can be used to determine the actual beam efficiency. Since beam efficiency is so easily measured, with its measurement embedded into almost every GBT project already, the theoretical beam efficiency is useful for planning purposes much more than it should be used for the actual calibration of astronomical data.

There's absolutely nothing new here since I'm merely performing algebra on equations found in all the standard calibration papers (e.g., Baars, 1973; Goldsmith, 1987, 2002; Rohlfs and Wilson, 2006; etc.). All I have done is taken the liberty to convert notations into those we use most often in Green Bank. In these papers, beam efficiency (η_{MB}) is defined as the percentage of received power which enters the main beam, up to the first nulls in the beam pattern, from a uniform source that covers 4π stereradians. All define it as the ratio:

$$\eta_{\mathrm{MB}} = \frac{\Omega_{\mathrm{MB}}}{\Omega_{\mathrm{A}}}$$

where Ω_A is the antenna solid angle (integral of the normalized power pattern over 4π stereradians) and Ω_{MB} is the solid angle of the main beam up to the first nulls in the beam pattern (integral of the normalized power pattern over the main beam).

Astronomer beware!!! The usefulness of beam efficiency is extremely limited due to the above technical definition. Except in some very rare and fortuitous experiments, the observed objects do not meet the criteria for which beam efficiencies can be used to derive physically relevant parameters.

The antenna theorem gives:

$$\Omega_{\mathrm{A}} = \frac{\eta_{\mathrm{R}}\lambda^2}{A_e} = \frac{4\eta_{\mathrm{R}}\lambda^2}{\eta_{\mathrm{A}}\pi D^2}.$$

D is the dish diameter, λ the observing wavelength, η_A is the aperture efficiency (ratio of the absorption area of the dish to its physical area), and η_R is the radiation efficiency, which many authors at this point assume equals one, which is a good approximation for telescopes like the GBT. Thus:

$$\eta_{MB}/\eta_A = \Omega_{\mathrm{MB}}\frac{\pi D^2}{4\eta_{\mathrm{R}}\lambda^2}.$$

One can derive this ratio from any of the GBT pointing measurements by determining the approximate angular offset at which the measured beam profile meets the baseline. If the beam can be assumed to be symmetric, then just one of the two orthogonal directions in the pointing is needed. Otherwise, one can assume an elliptical beam and use the measured widths in the two orthogonal directions as the major and minor axes of an equivalent ellipse.

Since it may be difficult to determine the angular distance at which the beam obtains its first null (e.g., the source is weak or one is confusion limited), one can assume the beam is Gaussian and use the measured full-width, half maximum beam width (FWHM) as a proxy for the width at the first null. Various authors give the following approximation:

$$\Omega_{\rm MB} \approx \frac{\pi}{4\ln 2} \cdot \theta_{\rm X}^{\rm FWHM} \cdot \theta_{\rm Y}^{\rm FWHM}.$$

Here, θ is the width in the two orthogonal directions of our pointings. In most cases, especially at low frequencies, these can be considered the same. Following on with the algebra:

$$\eta_{MB}/\eta_A \approx \frac{\pi}{4\ln 2} \cdot \theta_{\rm X}^{\rm FWHM} \cdot \theta_{\rm Y}^{\rm FWHM} \cdot \frac{\pi D^2}{4\eta_{\rm R}\lambda^2} = 0.8899 \cdot \theta_{\rm X}^{\rm FWHM} \cdot \theta_{\rm Y}^{\rm FWHM} \cdot \frac{D^2}{\eta_{\rm R}\lambda^2}.$$

For the GBT, the standard published relationship is θ(radians) $\approx 1.23\times10^{-4}\,\lambda$(cm), though one must realize that the multiplicative factor depends upon the details of the feed illuminations, which differs not only from receiver to receiver but also within each receiver's band.

Alternatively, if estimates or measurements of the beam width are unavailable, one can determine an approximate theoretical beam width from a known feed illumination taper using:

$$\theta^{FWHM} \approx [1.02 + 0.0135 \cdot FeedTaper(dB)]\frac{\lambda}{D}$$

(Goldsmith, 1987, 2002), which assumes a feed illumination with a Gaussian taper, as is the case for most GBT receivers. Thus,

$$\eta_{MB}/\eta_A \approx 0.8899 \cdot [1.02 + 0.0135 \cdot FeedTaper(dB)]^2/\eta_{\rm R}.$$

Due to the somewhat strong dependence on feed taper, the value of this ratio can vary substantially. For example, since standard feed tapers for the GBT can vary from 10 to 17 dB, and assuming η_R=1, η_{MB}/η_A can range from ≈1.19 to 1.39. Our 'standard' taper of 15 dB gives $\eta_{MB}/\eta_A \approx 1.33$. For the 43-m when it was using hybrid feeds, the measured ratio was ≈1.38.

References

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